THE FEBRUARY MEETING IN NEW YORK

The three hundred fifty-seventh meeting of the American Mathematical Society was held at Columbia University on Saturday, February 25, 1939. The attendance included the following one hundred ninety-six members of the Society:

C. R. Adams, R. B. Adams, R. P. Agnew, Leon Alaoglu, C. B. Allendoerfer, R. G. Archibald, Herbert E. Arnold, R. P. Bailey, S. F. Barber, Sr. M. A. Beamish, M. F. Becker, T. J. Benac, A. C. Berry, E. E. Betz, G. D. Birkhoff, W. Z. Birnbaum, Archie Blake, Gertrude Blanch, J. H. Blau, H. F. Bohnenblust, Samuel Borofsky, Joseph Bowden, C. B. Boyer, H. W. Brinkmann, A. B. Brown, J. H. Bushey, S. S. Cairns, A. D. Campbell, W. B. Campbell, M. E. Carlen, Alonzo Church, A. B. Coble, T. F. Cope, Richard Courant, Ruth Crucet, H. B. Curry, M. D. Darkow, Norman Davidowitz, M. M. Day, J. J. DeCicco, Jesse Douglas, Arnold Dresden, Nelson Dunford, W. D. Duthie, J. E. Eaton, H. T. Engstrom, Paul Erdös, J. M. Feld, Aaron Fialkow, T. S. Fiske, D. A. Flanders, M. C. Foster, R. M. Foster, Philip Franklin, K. O. Friedrichs, T. C. Fry, G. N. Garrison, J. J. Gergen, B. P. Gill, R. L. Gilman, M. C. Graustein, W. C. Graustein, M. C. Gray, M. B. Greenebaum, Laura Guggenbühl, M. B. Haberzetle, D. W. Hall, N. A. Hall, Israel Halperin, D. C. Harkin, Philip Hartman, G. A. Hedlund, Fritz Herzog, E. H. C. Hildebrandt, Banesh Hoffmann, Lulu Hofmann, M. A. Holly, G. M. Hopper, Harold Hotelling, S. E. Hotelling, E. M. Hull, R. P. Isaacs, R. A. Johnson, A. W. Jones, B. W. Jones, Mark Kac, E. R. van Kampen, Edward Kasner, L. S. Kennison, B. F. Kimball, J. R. Kline, L. A. Knowler, E. R. Kolchin, Solomon Lefschetz, B. A. Lengyel, Howard Levi, Marie Litzinger, E. R. Lorch, A. N. Lowan, R. S. McKee, Brockway McMillan, L. A. MacColl, H. F. MacNeish, Dorothy Manning, W. T. Martin, A. E. Meder, H. L. Mintzer, J. A. Mira, Virginia Modesitt, Deane Montgomery, Lillian Moore, A. P. Morse, Marston Morse, H. T. Muhly, D. C. Murdoch, F. J. Murray, Tadasi Nakayama, D. S. Nathan, John von Neumann, Oystein Ore, J. C. Oxtoby, C. K. Payne, B. J. Pettis, Everett Pitcher, E. L. Post, Walter Prenowitz, R. G. Putnam, H. A. Rademacher, H. W. Reddick, M. S. Rees, R. W. Rempfer, H. J. Riblet, Moses Richardson, R. G. D. Richardson, H. E. Robbins, J. H. Roberts, M. S. Robertson, S. L. Robinson, Benjamin Rosenbaum, R. A. Rosenbaum, Barkley Rosser, S. G. Roth, O. K. Sagen, Arthur Sard, Samuel Saslaw, O. F. G. Schilling, Abraham Schwartz, Hyman Serbin, I. M. Sheffer, Jacob Sherman, Max Shiffman, L. G. Simons, James Singer, Abraham Sinkov, P. A. Smith, Virgil Snyder, V. E. Spencer, M. H. Stone, W. C. Strodt, D. J. Struik, R. R. Struik, M. M. Sullivan, C. S. Sutton, J. L. Synge, J. M. Thomas, C. B. Tompkins, C. C. Torrance, E. M. Torrance, A. W. Tucker, J. W. Tukey, Annita Tuller, S. M. Ulam, Oswald Veblen, R. J. Walker, Henry Wallman, J. V. Wehausen, Louis Weisner, M. E. Wells, Hermann Weyl, A. P. Wheeler, O. L. Wheeler, H. S. White, A. L. Whiteman, Hassler Whitney, G. T. Whyburn, D. V. Widder, Norbert Wiener, C. R. Wilson, H. P. Wirth, Jacob Wolfowitz, H. A. Wood, H. N. Wright, W. M. Wynne, Marie York, Leo Zippin, H. S. Zuckerman.

The meeting opened Saturday morning in three sessions: Algebra and Number Theory, Professor M. H. Stone presiding; Analysis,

Vice President C. R. Adams presiding; and Geometry and Topology, Professors J. R. Kline and A. D. Campbell presiding. At the afternoon general session, at which Vice President J. F. Ritt presided, Professor R. P. Agnew gave an address entitled *Properties of generalized definitions of limit.*

A meeting of the Council was held on Saturday at 12:30 P.M. in the Faculty Club of Columbia University, Vice President Ritt in the chair. The election of the following four persons to membership in the Society was announced by the Secretary:

Professor Joseph T. O'Callahan, Holy Cross College;
Dr. Henry William Steinhaus, Equitable Life Assurance Society of the U.S., New York, N.Y.;
Mr. Robert R. Stoll, Rensselaer Polytechnic Institute;
Mr. Benjamin Joseph Tepletsky, Ohio State University.

It was reported that the following had been elected as nominees on the Institutional Memberships of the various institutions and departments:

University of California: Messrs. Charles Amos Heyes, Arthur Elmore Marston, and William Francis Whitmore.
Columbia University: Miss Ruth Crucet, Messrs. Rufus Philip Isaacs, Donald Percy Ling, and Harry Sohn, and Mrs. Marie York.
Duke University: Messrs. Paul Wilner Gilbert, Donald M. Seward, and Luther Irwin Wade.
Division of Mathematics, Harvard University: Messrs. Francis Parkman Byerly, Richard Frederick Clippinger, Thomas Jefferson Coolidge, Clinton Hoadley Crane, Nathan Hayward, Lynn Harold Loomis, Douglas Tillman McClay, George Putnam, and Abraham Spitzbart.
Haverford College: Mr. Philip Martin Whitman.
Iowa State College: Mr. Donald Hill Rock.
The Johns Hopkins University: Dr. Mark Kac.
Lehigh University: Messrs. Francis Ernest Carner and William Reagle Transue.
Stanford University: Miss Rhoda Manning and Mr. Robert Wayman Rector.
Wesleyan University: Mr. Arthur Southcomb Day.
Western and Southern Life Insurance Company: Professor Victor Cyril Stechschulte.
University of Wisconsin: Messrs. James Henry Bell, Edwin Louis Crow, Vladimir Morkovin, Homer Edward Newell, Jr., Robert Dickerson Specht, Bonnie Madison Stewart, and James Audley Ward.
Department of Mathematics, Yale University: Mr. Theodore Joseph Benac.

The following appointments by President Evans and Vice President Ritt were reported: as representative of the Society at the Centennial Celebration of Duke University to be held at Durham, North Carolina, on April 21, 22, and 23, Professor E. T. Browne; as a committee to make recommendations to the Council regarding the use of a bequest from the estate of E. W. Brown, Professors W. R. Long-

ley (chairman), Virgil Snyder, and K. P. Williams; as a Committee on Nomination of Officers and Members of the Council for 1940, Professors Dunham Jackson (chairman), G. A. Bliss, D. J. Struik, T. Y. Thomas, and Marie Weiss; as a committee to consider the location of the Annual Meeting of 1940, Professors W. L. Ayres (chairman), E. T. Browne, and Philip Franklin; as a committee to consider freer methods of nominating and electing members-at-large in the Council, Dean L. P. Eisenhart (chairman), Professors C. R. Adams, W. C. Graustein, W. T. Martin, and A. D. Michal; as a committee on the place of meetings, Professors E. W. Chittenden (chairman), W. B. Carver, W. L. Hart, E. J. McShane, and H. B. Phillips; as a committee on the use of the accrued income from special funds, Professors T. H. Hildebrandt (chairman), Arnold Dresden, W. B. Fite, J. L. Walsh, and Anna Pell Wheeler.

It was announced that Professor Theodore von Kármán has accepted the Society's invitation to give the fifteenth Josiah Willard Gibbs Lecture at Columbus.

In place of the usual February and April meetings, only one spring meeting will be held in 1940. An invitation was received from George Washington University for this meeting.

The Committee on the Abstract Journal reported progress.

Titles and cross references to the abstracts of the papers read at this meeting follow below. Papers whose abstract numbers are followed by the letter *t* were read by title. The papers numbered 1 to 8 were read before the section for Algebra and Number Theory; those numbered 9 to 15 before the section for Analysis; those numbered 16 to 22 before the section for Geometry and Topology; papers numbered 23 to 51 were read by title. Paper 5 was read by Dr. Murdoch, paper 10 by Mr. Wolfowitz, paper 13 by Mr. McMillan, paper 18 by Dr. Schilling, and paper 20 by Professor Kasner. Dr. Abraham Wald was introduced by Professor Harold Hotelling, Dr. Mark Kac by Professor Aurel Wintner, Mr. R. P. Isaacs and Miss Annette Vassell by Professor Edward Kasner, Miss Harlan C. Miller and Mr. R. L. Swain by Professor R. L. Moore, and Professor Harald Cramér by Professor Einar Hille.

1. J. H. Blau: *A characterization of algebraic fields of prime characteristic.* (Abstract 45-3-109.)

2. H. J. Riblet: *Symmetric differential expressions.* (Abstract 45-3-125.)

3. Barkley Rosser: *On the first case of Fermat's last theorem.* (Abstract 45-1-96.)

4. Abraham Sinkov: *A note on a paper by J. A. Todd.* (Abstract 45-3-126.)

5. D. C. Murdoch and Oystein Ore: *Ideal theory in non-associative rings.* (Abstract 45-3-122.)

6. Tadasi Nakayama: *On Frobeniusean algebras.* I. (Abstract 45-3-123.)

7. Oystein Ore: *Chain properties in finite groups.* (Abstract 45-3-124.)

8. J. E. Eaton: *Associative multiplicative systems.* (Abstract 45-3-111.)

9. Abraham Wald: *Lower and upper limits of a distribution function determined by moments and inequalities satisfied by moments.* (Abstract 45-3-150.)

10. Abraham Wald and Jacob Wolfowitz: *Confidence limits for continuous distribution functions.* (Abstract 45-3-128.)

11. Philip Hartman: *An asymptotic formula for exponential integrals.* (Abstract 45-3-114.)

12. Mark Kac: *Statistics and power series with gaps.* (Abstract 45-3-117.)

13. Norbert Wiener and Brockway McMillan: *A new method in statistical mechanics.* (Abstract 45-3-133.)

14. Vivian E. Spencer: *Extensions of theorems of Markoff and Krein.* (Abstract 45-1-97.)

15. E. R. Kolchin: *On the basis theorem for systems of differential polynomials.* (Abstract 45-3-119.)

16. R. P. Isaacs: *A geometric interpretation of the difference quotient of polygenic functions.* (Abstract 45-1-90.)

17. S. S. Cairns: *Planes transversal to polyhedral manifolds.* (Abstract 45-3-110.)

18. Saunders MacLane and O. F. G. Schilling: *Zero-dimensional branches of rank one on algebraic varieties.* (Abstract 45-3-120.)

19. S. M. Ulam: *On ϵ-isomorphic transformations.* Preliminary report. (Abstract 45-3-127.)

20. Edward Kasner and J. J. DeCicco: *Curvature element transformations which preserve integrable fields.* (Abstract 45-3-118.)

21. J. J. DeCicco: *The polygenic functions whose associated series are equiparallel series.* (Abstract 45-1-84.)

22. J. M. Feld: *Conformal and inversive invariants of plane curves and horn angles.* (Abstract 45-3-112.)

23. Garrett Birkhoff: *The mean ergodic theorem.* (Abstract 45-1-77-*t*.)

24. Garrett Birkhoff and Morgan Ward: *A characterization of Boolean algebras.* (Abstract 45-1-78-*t*.)

25. O. K. Bower: *Paradoxes involving mathematical expectation.* (Abstract 45-1-80-*t*.)

26. O. K. Bower: *The equation $f(x+y)=f(x)f(y)$ and a system of functional equations depending upon its solution.* (Abstract 45-1-81-*t*.)

27. George Comenetz: *The limit of the ratio of arc to chord for a space curve.* (Abstract 45-1-82-*t*.)

28. J. J. DeCicco: *The analogue of the Moebius group of circular transformations in the associated Kasner plane.* (Abstract 45-1-83-*t*.)

29. F. C. Gentry: *Cremona involutions determined by a pencil of surfaces.* (Abstract 45-3-113-*t*.)

30. Einar Hille: *Remarks concerning group spaces and vector spaces.* (Abstract 45-1-89-*t*.)

31. Charles Hopkins: *An extension of a theorem of Remak.* (Abstract 45-3-115-*t*.)

32. Nathan Jacobson: *An application of E. H. Moore's determinant of a hermitian matrix.* (Abstract 45-3-116-*t*.)

33. B. O. Koopman: *The axioms and algebra of intuitive probability.* (Abstract 45-1-92-*t*.)

34. E. H. Larguier: *On the foundations of mathematics.* (Abstract 45-1-93-*t*.)

35. Harlan C. Miller: *Some methods of characterizing certain atriodic continua.* (Abstract 45-3-121-*t*.)

36. Barkley Rosser: *Definition by induction in Quine's "New foundations for mathematical logic."* (Abstract 45-1-95-*t*.)

37. R. H. Sorgenfrey: *A theorem on atriodic continua.* (Abstract 45-1-63-*t*.)

38. R. L. Swain: *On continua obtained from sequences of simple chains of connected regions.* (Abstract 45-1-98-*t*.)

39. Annette Vassell: *Sectional families of curves.* (Abstract 45-1-99-*t*.)

40. A. D. Wallace: *Non-boundary sets.* (Abstract 45-3-129-*t*.)

41. A. D. Wallace: *Some characterizations of interior transformations.* (Abstract 45-3-130-*t*.)

42. J. L. Walsh: *On the circles of curvature of the images of circles under a conformal map.* (Abstract 45-3-131-*t*.)

43. Morgan Ward: *A note on divisibility sequences.* (Abstract 45-1-101-*t*.)

44. Morgan Ward: *Ring homomorphisms which are also lattice homomorphisms.* (Abstract 45-1-104-*t*.)

45. Morgan Ward and R. P. Dilworth: *Lattice theory of ova.* (Abstract 45-1-107-*t*.)

46. Hassler Whitney: *On products in a complex.* II. (Abstract 45-3-132-*t*.)

47. Harald Cramér: *On the representation of a function by certain Fourier integrals.* (Abstract 45-5-170-*t*.)

48. H. T. Engstrom: *A note on fundamental systems of symmetric functions.* (Abstract 45-3-137-*t*.)

49. D. H. Lehmer: *On the remainders and convergence of the series for the partition function.* (Abstract 45-3-140-*t*.)

50. Max Zorn: *Continuous groups and Schwarz' lemma.* (Abstract 45-3-151-*t*.)

51. Mark Kac and E. R. van Kampen: *Circular equidistribution and statistical independence.* (Abstract 45-5-194-*t*.)

T. R. HOLLCROFT,
Associate Secretary

ERNEST WILLIAM BROWN—IN MEMORIAM*

Ernest William Brown was born at Hull, England, on November 29, 1866, the only son of a prosperous farmer. He went to school at East Riding College where he showed such an aptitude for mathematics that it was decided that he was to go on to Cambridge. Here he was graduated in 1887 as sixth wrangler, and was elected a fellow. In 1891 he received a call from Haverford College. There he remained till 1907, when he was appointed professor of mathematics at Yale University, in a sense as successor to Willard Gibbs. When the Sterling professorships were established at Yale in 1921, he was among the first to receive one of these appointments. In 1931 he was elected first incumbent of the Willard Gibbs professorship. The next year he retired because of ill health and because he found his teaching duties somewhat irksome. His retirement did not mean, however, the cessation of his activities.

At Cambridge the instructor who most influenced him was George Darwin. On his advice young Brown turned to a study of Hill's great memoir of 1877 on the lunar theory. He became more and more deeply interested in this subject, and when he went to Haverford he had decided to undertake the enormous task of computing the moon's orbit on this theory. This was to occupy him to the practical exclusion of every other research for the next quarter of a century; indeed he was busy on one phase or another of the lunar problem throughout the remainder of his life. So thoroughly and minutely did he carry out this work that it is difficult to imagine that it will need revision in any form for many decades to come. In the progress of this work he acquired a facility for throwing mathematical expressions into easily computable form that has rarely been equalled.

His chief object was to ascertain whether the attractions of the known bodies in the solar system would account for the moon's motion. He found that in two respects they apparently fail to do so. The first is with regard to the "secular acceleration" discovered by Halley two centuries earlier. Part of this turns out to be a secular *retardation* of the rate of rotation of the earth; two English geophysicists have since shown that tidal friction in shallow channels (the Irish Sea and Behring Strait, for example) is of the right order to correspond to the observed retardation. The other respect in which gravitation fails to account for the moon's motion is the presence of "fluctuations" (so called by Newcomb, who discovered them), alternate retardations and accelerations in the moon's place in periods between two and four decades. Brown showed that these too are in all probability irregularities in the rotation of the earth, though their origin is still mysterious.

Once the moon was out of the way, Brown went on with great success to other difficult problems in celestial mechanics, notably those of the orbits of the eighth satellite of Jupiter and of the Trojan group of asteroids, objects that form with Jupiter and the sun an approximate equilateral triangle. During the last two or three years of his life he turned his attention, again with happy results, to the stellar case of three bodies.

Although he was not very fond of teaching, Brown performed these duties with

* In R. C. Archibald's *Semicentennial History of the American Mathematical Society*, p. 173–183, there is an article on Professor Brown, containing a bibliography with one hundred eighty-one entries. Since this was finished after Professor Brown's death, it may be regarded as complete. In the present notice, unnecessary duplication of data given by Archibald has been avoided.

distinction. He did not take the trouble to prepare his lectures very carefully in advance, and as a result he was not infrequently "stuck," but only for a moment—it was delightful to see how quickly his mind worked to extricate itself, as it invariably and promptly did. The process was a most instructive one for his pupils to witness.

He was greatly interested in university administration and his advice in such matters was highly esteemed at Yale and elsewhere.

His interests in life were many and varied. In his youth he devoted much time to mountain climbing and to rowing. He was a good performer on the piano until a few years before his death. He was no mean contestant at chess. He was an authority on Gilbert and Sullivan's operas, on the Bab Ballads, and on Lewis Carroll's poems, all of which he could recite from memory without hesitation.

Like some of his predecessors in celestial mechanics, Brown never married. His household was managed for many years by his maiden sister, Mildred, his junior by two years. She made his comfort the chief object of her life and succeeded in thoroughly spoiling him. She died some three or four years before he did, and thereafter his nearest relatives were a sister and her children in New Zealand. Throughout his life he set high score upon social contacts and he had a knack for making deep and life long friendships. He was a constant attendant upon scientific meetings but did not by any means attend all the sessions for the reading of papers; undoubtedly his chief object in going was to maintain or renew his friendships among his colleagues.

Brown had been a severe sufferer from bronchial troubles since his early manhood, probably as a result of his activities as a rower. In 1932, just before his retirement, he endured an attack of intestinal ulcers for which he refused to take the usual treatment. "Don't try to prolong my life," he told his physician, "just make me as comfortable as you can as long as I am here." Strange to say, this ailment cured itself, and six more years of mental vigor and productiveness were granted to him; but he had a hard time during these years and he died from sheer exhaustion on July 22, 1938, in his seventy-second year. His exit removes from the stage the last of a great group of workers in celestial mechanics upon whose like we shall not soon look again.

FRANK SCHLESINGER

SHORTER NOTICES

Les Fondements Expérimentaux de L'Analyse Mathématique des Faits Statistiques. By Georges Hostelet. (Actualités Scientifiques et Industrielles, no. 552.) Paris, Hermann, 1937. 71 pp.

In the first division the author points out that theorists, who desire to produce theory from a few axioms by purely logical reasoning, often are lead to conclusions which are experimentally incorrect. He defines the empirical-experimental method, the deductive-abstract method, and the abstract-experimental method and states that they are used in the study of statistics. Statistical facts are classified as singular, as aggregative, and as those connected with species and typical groups. The author gives the meaning of the law of large numbers, discusses the theory of criteria, defines objective and subjective judgments, and mentions Laplace's attempts to apply mathematical probability to human witnessing. Indexes are mentioned. Statistics discovers genera with many species.

The second chapter contains a brief discussion of the Gaussian Law, which the author calls the binomial frequency law, and a statement that this law arose from the axiom concerning the arithmetic mean being the best value of the quantity measured. It is pointed out that the arithmetic mean should come from the law instead of the law coming from the mean.

The third division, which is very philosophical, treats of the meaning and problems of causality, defines in many details absolute independence and dependence of cause and effect, the principle of scientific causality, experimental induction, the abstract experimental method, a conditional experimental law, experimental reasoning, experimental statistics, discriminate and abstract analyses, and states how the relations between mathematical and experimental statistics are used in examining mathematical statistics. The author concludes this division by giving the equation and some of the properties of the correlation frequency surface pertaining to two variables.

The first chapter of the second part presents definitions of the probable, logical, and mathematical probability. The author states that empirical-intuitive ideas form the basis of the calculation of probability, explains in some detail the fundamentals which enter into games of hazard, and relates how *a posteriori* probability has created confusion with regard to causal and non-causal events. He states that the accepted ideas concerning hazard came from the natural order of things and from human intelligence. The notion of hazard can be explained through the experimental notions of causality and finality.

The next chapter treats of a critical study of the theory of contingency and correlation from the point of view of the scientific methodology of investigation. This contains most of the meat of the book, for here the author discusses Karl Pearson's ideas of contingency and correlation and takes issue with Pearson's views concerning correlation, especially with the degree of association. He concludes this section by stating that he has never found a precise and adequate experimental definition of the degree of association or correlation, and because of this fact all formulas furnish only arbitrary values, or values without experimental foundation.

The last chapter contains an excellent summary of what is in the book.

W. D. Baten

Theoretical Mechanics. A Vectorial Treatment. By C. J. Coe. New York, Macmillan, 1938. 13+555 pp.

This treatise on classical mechanics contains in fourteen chapters 534 pages of text. In the first chapter the author discusses briefly the divisions, postulates, and validity of mechanics. The second of these is more extensively considered in Chapter XII on the General Principles of Mechanics, such as: Lagrange's Equations, Hamilton's Principle, Least Action, and Gauss' Least Constraint.

The second chapter deals with Rectilinear Motion of a Particle making use of the customary scalar calculus method. This is followed by a long chapter on the theory of vectors through the double cross-product and simple differentiation and integration of vectors. More of the calculus of vectors is presented in Chapter XIII as a foundation for the proof of Stokes' and Green's Theorems and a final chapter on Potential Theory.

From the second chapter onward the vector approach is used consistently and beautifully effectively in presenting the theory by classifying vectors as free, sliding, or attached. However, in applications, the solution is frequently effected principally by scalar calculus. Many of the more advanced theorems of classical mechanics are stated and proved by vector analysis. The particular order of presentation postpones a treatment of statics until Chapter VII, where it is considered to be a special case of zero acceleration and velocity.

Chapters VII to XI discuss: Statics of a Particle and of a Rigid Body, Statics of a Flexible Cord, Principle of Virtual Work, Kinetics of a Particle and Kinetics of a Rigid Body. Some topics of considerable technical importance, such as the elastic catenary, impact, and cases of chains involving variable momentum are not included.

In the introduction the author states that "this book presents sufficient material for an introductory and advanced course each running through the year." It is an excellent procedure for a serious student of mechanics to take a two-year course such as this book provides. He will thus obtain a knowledge of vector analysis and at the same time acquire considerable facility in its use in the field of mechanics. Unfortunately, in most undergraduate curricula there is not sufficient time allowed for a student of mechanics to follow this procedure.

Apparently the trend in technical schools is rather away from heavy courses in mechanics than towards them. The teacher finds himself forced to do the best he can with the usual preparation in calculus, no matter how desirable a knowledge of vector analysis may be in a study of analytic mechanics. For this reason, the reviewer doubts that Professor Coe's treatise will find broad use in undergraduate courses. However, for advanced students wishing to combine a study of vector analysis with mechanics, or for students specializing in mechanics, it appears to be a very fine presentation.

J. B. Reynolds

Introduction to Bessel Functions. By F. Bowman. London, Longmans, Green, 1938. 135 pp.

Chapter I (19 pages) gives elementary properties of Bessel functions of order 0, and comments (without proofs) on expansions of "ordinary functions of mathematical physics" in terms of orthogonal Bessel functions of order 0. Chapter II (21 pages) derives some partial differential equations and sets up and solves some boundary-value problems. These problems are well chosen to exhibit standard methods of solution. The problems are not carefully phrased in that nothing is said as to whether the differential equation is to be satisfied at points on the boundary, or indeed whether

there is to be any condition whatever (such as continuity, etc.) connecting values in the interior with values on the boundary. The solutions are formal; for example, on page 39 we find the assertion that a function v defined by an infinite series $v=\sum A_n v_n$ (the A's being undetermined arbitrary constants and the v's being functions of r and t) satisfies a differential equation and the condition $\lim_{t\to\infty} v(r, t)=0$ "since this is true of every term."

Chapter III (16 pages) discusses briefly Bessel functions of order 0 when the argument is pure imaginary, and gives applications to problems in alternating currents. Chapters IV and V (30 pages) give definite integrals and asymptotic expansions involving Bessel functions of order 0. Chapter VI (35 pages) gives fundamental properties of Bessel functions of real orders, and finally Chapter VII (13 pages) gives applications of them.

The book contains about 150 exercises and problems, many of which consist of several parts and most of which require proofs of identities involving series or integrals. The emphasis in the book is on formulas and identities rather than on rigorous methods of obtaining them. The reviewer feels that the book would be made more useful if numbers of displayed formulas were placed before rather than after the formulas, and the space after the formulas were used to specify the ranges of the parameters for which the formulas hold. The choice of notations and printing is good except for an annoying similarity of two microscopically different Y's which denote different Bessel functions. Finally, the reviewer must report (for the attention of authors and publishers) that the binding of his copy of the book cracked badly in spite of a careful attempt to open the book without tearing it apart.

R. P. AGNEW

Advanced Analytic Geometry. By A. D. Campbell. New York, Wiley; London, Chapman and Hall, 1938. 10+310 pp.

Professor Campbell envisages a student who had a rudimentary course in plane analytic geometry in which oblique axes have not been mentioned. The algebraic equipment of the student may exceed somewhat the usual course in College Algebra of our American schools, say, in the matter of determinants, but he cannot be relied upon to be familiar with Sylvester's method of elimination (p. 93). His knowledge of derivatives hardly goes beyond the definition of the term.

This student Professor Campbell undertakes to "introduce to the analytic side of projective geometry." The author realizes that he will have to confront his student with a vast number of new ideas, both analytical and geometrical, and that the student may have difficulties in assimilating new concepts coming in such rapid succession. To meet the situation the author deliberately sets out to remove from the path of the learner every obstacle that can possibly be removed. He begins by picking out a considerable number of topics which are usually dealt with, or made use of, in analytic projective geometry, but which can be treated independently of that subject. He puts this material in the front part of the book, to form preliminaries, or an introduction, to the subject proper. Thus he discusses affine geometry, linear transformations, groups, anharmonic ratio, families of conics, etc. Then he develops each topic gradually, with plenty of examples, without sudden jumps, and with constant regard to the mathematical equipment of the learner. By the time this part of the task is done, the author has not only completed about half of his book, but has produced something that constitutes a rounded whole in itself, and well worth the while of anyone who would drop the subject right there.

The second part of the book takes up the subject of projective analytic geometry, by discussing the usual topics, such as the triangle of reference, homogeneous coordinates, duality, the line at infinity, etc. Some of the topics already considered in the first part are developed further. All this is written with the same preoccupation for clarity and simplicity as the first part. The student will find many judicious remarks, many striking "asides" which cannot but interest him. Constant appeal is made to the student's geometric intuition. Synthetic methods are often made use of, and even synthetic proofs are given, when this procedure seems to simplify and expedite matters. Exercises are numerous throughout the book and often used to supply details of proofs and to clarify the text generally.

It is possible to raise the question whether the author, by making the presentation too easy, has not deprived the student of his birthright to come to grip with difficulties. This reviewer, for one, shares the author's view that such fears are futile. The difficulties inherent in the subject matter discussed, and in Mathematics, in general, are a sufficient guaranty that the learner will have to put forward his best efforts, if he is to get anywhere, and it is not necessary to pile up difficulties of presentation, just for the pleasure of having them there. It is more reasonable to smoothen the learner's path when possible and thus enable him to reach more quickly a higher level in his studies.

However, the reviewer does not share the author's view that it is always advisable to assume as meager a preparation on the part of the student as possible. The author followed a custom which has attained the dignity and the rigidity of a dogma. This "starting from scratch" is a burdensome procedure that impedes progress, and is often unnecessary.

The book as a whole is well thought out, well planned, and well written.

N. A. COURT

Introductory Quantum Mechanics. By V. Rojansky. New York, Prentice-Hall, 1938. 529 pp.

The prerequisites for a study of this book are "the elements of calculus and of ordinary differential equations, and a recognition of the failure of classical mechanics in the domain of atomic physics." The book is for those who are willing to devote the necessary time for getting a sound working knowledge of the subject. The emphasis is on ideas and fundamentals. Thus the problems are selected for mathematical simplicity and for their appropriateness in illustrating the theory. The free particle and the harmonic oscillator are most often used.

Mathematical and physical difficulties receive separate treatment. The mathematics of operators, eigenfunctions and eigenvalues, and matrices is treated fully enough for the comfortable reading of the physical part. The book closes with chapters on the states of the normal hydrogen atom, electron spin, and Dirac's theory of the electron.

It seems to the reviewer that the author has succeeded in his aim to introduce the reader to Quantum Mechanics, and that most readers would save time and get better results by working through this book before going to a more "comprehensive and critical survey of the theory, or a study of its applications" for which it is an introduction. The coherence of the book is to be praised. The author has made the subject his own and has not written a compilation.

K. W. LAMSON

NOTES

The James Scott Prize of the Royal Society of Edinburgh was presented to Professor P. A. M. Dirac of the University of Cambridge at a meeting of the Society on February 6.

The James Alfred Ewing Medal has been awarded to Professor A. H. Gibson of the University of Manchester.

The Board of Directors of the Research Corporation of New York has awarded $2,500 to Dr. Vannevar Bush, president of the Carnegie Institution of Washington, in recognition of his invention of mathematical computing machines.

Professor Harlow Shapley, director of the Harvard Observatory, has received the Bruce Medal of the Astronomical Society of the Pacific.

Professor Arthur Conway of University College, Dublin, has been appointed a member of the Pontifical Academy of Sciences.

Dr. William Findlay, professor of mathematics at the University of Toronto, recently gave an address as president of the Royal Society of Canada. His subject was *Mathematics and astronomy.*

The following mathematicians were starred for the first time in the sixth edition of American Men of Science: L. V. Ahlfors, Harvard University (now at the University of Helsingfors); Garrett Birkhoff, Harvard University; Salomon Bochner, Princeton University; Alonzo Church, Princeton University; Richard Courant, New York University; H. T. Davis, Indiana University; Harold Hotelling, Columbia University; B. O. Koopman, Columbia University; E. J. McShane, University of Virginia; A. D. Michal, California Institute of Technology; H. A. Rademacher, University of Pennsylvania; H. P. Robertson, Princeton University; Otto Szász, University of Cincinnati; Gabor Szegö, Stanford University; J. M. Thomas, Duke University; W. J. Trjitzinsky, University of Illinois; Hermann Weyl, Princeton University; Hassler Whitney, Harvard University; Aurel Wintner, Johns Hopkins University; Oscar Zariski, Johns Hopkins University.

The one hundredth anniversary of the birth of Josiah Willard Gibbs was commemorated by Yale University on February 16.

Professor H. F. Blichfeldt of Stanford University has retired with the title emeritus.

Professor J. A. Eiesland of West Virginia University has been given the title emeritus.

Professor Raymond Pearl of Johns Hopkins University was elected president of the American Statistical Association at its recent meeting in Detroit.

Professor H. N. Russell, director of the Princeton University Observatory, has been elected correspondent of the Section of Astronomy of the Paris Academy of Sciences in succession to the late Professor E. W. Brown.

Mr. H. C. Trimble has been awarded a General Education Board Fellowship.

Dr. S. Goldstein, fellow and lecturer in mathematics at St. John's College, Cambridge, is spending this term at the California Institute of Technology.

A project for the Computation of Mathematical Tables, sponsored by Dr. L. J. Briggs, Director of the National Bureau of Standards, is being conducted by the Works Progress Administration for the City of New York. Three tables are completed, five are being computed, and seven others are under consideration. The directors of the project desire to hear from those conducting like work elsewhere and also from those who wish to offer suggestions for new tables. Communications may be addressed to Dr. A. N. Lowan, Chief Project Supervisor, 475 Tenth Avenue, New York City.

The following sixty-eight doctorates, with mathematics or mathematical physics as a major subject, were conferred during 1938 in universities in the United States and Canada; the major subject is mathematics unless otherwise specified. The university, month in which the degree was conferred, minor subject (other than mathematics), and the title of the dissertation are given in each case if available.

Leonidas Alaoglu, Chicago, August, *Weak topologies of normed linear spaces.*

G. E. Albert, Wisconsin, June, *Asymptotic forms of the generalized Legendre functions.*

Vera A. Ames, Bryn Mawr, June, *Linear equations with n parameters.*

Frank Ayres, Jr., Chicago, August, *A Cremona transformation generated by a pencil of surfaces.*

H. G. Barone, Cornell, June, *Limit points of sequences and their transforms by methods of summability.*

R. C. F. Bartels, Wisconsin, March, *Saint-Venant's flexure problem for a regular polygon.*

E. J. Bernet, Wisconsin, June, major in physics, minor in mathematics, *Gamma-rays from fluorine due to proton bombardment.*

Fannie W. Boyce, Chicago, August, *Certain types of nilpotent algebras.*

C. P. Brady, Chicago, August, *The minimum of a function of integrals in the calculus of variations.*

B. W. Brewer, Jr., Missouri, August, *Criterion for solvability by radicals in an algebraically perfect field.*

D. M. Brown, Illinois, February, minor in physics, *Arithmetics of rational generalized quaternion division algebras.*

C. R. Cassity, Illinois, June, minor in physics, *The maps determined by the principal curves associated with five and six points in the plane.*

C. E. Clark, Cornell, June, *Simultaneous invariants of a complex and subcomplex.*

J. J. DeCicco, Columbia, July, *The geometry of whirl series.*

F. L. Dennis, Illinois, June, minor in astronomy, *Some geometric applications of algebraic correspondences.*

J. M. Dobbie, Illinois, June, minor in physics, *A generalized Lambert series.*

C. H. Dowker, Princeton, June, *Mapping theorems in non-compact spaces.*

Rev. W. C. Doyle, St. Louis, June, minor in physics, *A further generalization of Lambert series.*

F. A. Ficken, Princeton, June, *Riemannian and affine differential geometry of product-spaces.*

Sister Mary Charlotte Fowler, Catholic University of America, June, minors in physics and education, *The discriminant of the sextic of double point parameters of the plane rational quartic curve.*

Sister Catherine Francis Galvin, Catholic University of America, June, minors in chemistry and physics, *Two geometrical representations of the symmetric correspondence, $C_{n,n}$, with their interrelations.*

B. H. Gere, Massachusetts Institute of Technology, June, minor in physics, *On lengths of curves and areas of surfaces.*

C. H. Graves, Chicago, March, *On certain surfaces in five-dimensional space.*

J. W. Green, California (Berkeley), May, I. *A property of harmonic functions in three variables.* II. *Harmonic functions in regions with multiple boundary points.*

Marion Greenebaum, Bryn Mawr, June, *The nonexistence of integral normal bases in certain algebraic fields.*

Mary B. Haberzetle, Chicago, June, *Two new universal Waring theorems.*

D. W. Hall, Virginia, June, *A decomposition of true cyclic elements.*

N. A. Hall, California Institute of Technology, June, minor in physics, I. *A formal expansion theory for functions of one or more variables.* II. *The number of representations function for binary quadratic forms.*

P. R. Halmos, Illinois, June, minor in philosophy, *Invariants of certain stochastic transformations. The mathematical theory of gambling systems.*

P. C. Hammer, Ohio State, September, *Projective geometries over a pseudo-field*

Philip Hartman, Johns Hopkins, June, *Mean motions and almost periodic functions.*

R. J. Havens, Wisconsin, June, major in physics, minor in mathematics, *The variation of the electric moment of carbon monoxide and nitric oxide with inter-nuclear distance.*

Hsien-Yü Hsü, Washington (St. Louis), June, minor in physics, *Certain integrals and infinite series involving ultraspherical polynomials and Bessel functions.*

I. F. Keeler, Iowa, August, minor in applied mathematics, *Properties of completely regular spaces.*

E. H. Krause, Wisconsin, June, major in physics, minor in mathematics, *The sensitized fluorescence of potassium.*

R. R. McDaniel, Cornell, June, *Approximation to algebraic numbers by means of periodic sequences of transformations on quadratic forms.*

W. G. Madow, Columbia, December, *Contributions to the theory of multivariate statistical analysis.*

P. T. Maker, Illinois, June, minor in philosophy, *Conditions on $U(X, Y)$ and $V(X, Y)$ necessary and sufficient for the regularity of $U+iV$.*

Margaret E. Mauch, Chicago, December, *Extensions of Waring's theorem on seventh powers.*

M. G. Moore, Illinois, February, minor in physics, *On expansions in series of exponential functions.*

D. L. Netzorg, Illinois, February, minor in physics, *Mechanical quadrature formulas and the distribution of zeros of orthogonal polynomials.*

I. M. Niven, Chicago, June, *A Waring problem.*

G. H. Peebles, Minnesota, June, minor in physics, *Some generalizations of the theory of orthogonal polynomials.*

D. T. Perkins, Yale, June, *Automorphisms of finite abelian groups.*

Sam Perlis, Chicago, August, *Maximal orders in rational cyclic algebras of composite degree.*

J. A. Pierce, Michigan, June, *A study of a universe of n finite populations with application to moment-function adjustments for group data.*

Henry Primakoff, New York, June, major in physics, minor in mathematics, *Relations between the second and higher order processes in the electron-neutrino field theory.*

Haim Reingold, Cincinnati, June, *Invariants of a system of linear homogeneous differential equations of the second order.*

Eric Reissner, Massachusetts Institute of Technology, June, minor in physics, *Contributions to the theory of elasticity of non-isotropic materials.*

R. W. Rempfer, Illinois, February, minor in physics, *Space Cremona groups of transformations of planar type.*

H. E. Robbins, Harvard, June, *On the classification of the maps of a two-complex into a space.*

L. D. Rodabaugh, Ohio State, March, *The solution of a certain linear partial differential equation of the first order.*

S. S. Saslaw, Massachusetts Institute of Technology, June, minor in physics, *Gap theorem on Fourier series.*

W. T. Scott, Rice Institute, June, *Studies on continued fractions and infinite products.*

Max Shiffman, New York, June, minor in physics, *The Plateau problem for minimal surfaces of arbitrary topological structure.*

D. R. Shreve, Illinois, June, minor in physics, *A certain class of algebraic hypersurfaces in projective space S_r invariant under the symmetric substitution group $G_{(r+2)!}$.*

Ruth G. Simond, Michigan, February, *Relations between certain continuous transformations of sets.*

C. E. Springer, Chicago, December, *Metric geometry of surfaces in four-dimensional space.*

R. C. Staley, Michigan, February, *Contributions to the study of non-normal difference equations of the second order.*

Morris Teles, October, Illinois, minor in philosophy, *Geodesic correspondence between Riemannian spaces.*

Leonard Tornheim, Chicago, August, *Integral sets of quaternion algebras over a function field.*

A. M. Turing, Princeton, June, *Systems of logic based on ordinals.*

F. E. Ulrich, Harvard, June, *The problem of type for a certain case of Riemann surfaces.*

J. V. Wehausen, Michigan, February, *Studies in linear topological spaces.*

E. L. Welker, Illinois, June, major in statistics, minor in education, *Correlation in heterogeneous populations.*

Audrey Wishard, Harvard, June, *Functions of bounded type.*

H. A. Wood, Massachusetts Institute of Technology, December, minor in mechanical engineering, *Superposition theorems and an inverse boundary value problem in the theory of heat flow.*

J. W. Wrench, Jr., Yale, June, *The derivation of arctangent relations.*

Dean G. D. Birkhoff of Harvard University has been appointed to a Walker-Ames Professorship of Mathematics at the University of Washington for the summer quarter of 1939.

Mr. G. E. Jones has been appointed to an assistant professorship at Louisiana Polytechnic Institute.

Dr. G. E. Uhlenbeck, formerly professor of theoretical physics at the University of Utrecht, has been appointed to a professorship at the University of Michigan.

Assistant Professor C. H. Vehse of the University of West Virginia has been promoted to an associate professorship.

Dr. G. P. Wadsworth of the Massachusetts Institute of Technology has been promoted to an assistant professorship.

Dr. Louise A. Wolf of the University of Wisconsin at Milwaukee has been promoted to an assistant professorship.

Assistant Professor E. C. Goldsworthy of the University of California at Berkeley is on leave during the spring term.

Dr. Leon Gropper, Dr. Jeanette Fox Keston, Dr. A. J. Maria, and Dr. W. J. Van Stockum have appointments at Brooklyn College.

The following appointments to instructorships are announced: Albany Academy: Mr. Karl Johannes; University of California (Berkeley): Dr. F. W. Dresch; Herzl Junior College, Chicago, Ill.: Mr. Leonard Tornheim; Hunter College: Dr. Madeline Levin; University of Tennessee: Mr. G. W. Whitehead; West Virginia University: Dr. Margaret Buchanan Cole.

The death of M. Louis Fabry on January 26, 1939, has been announced by the Paris Academy of Sciences.

The death of Principal Pandit Hemraj of Dayal Singh College, Lahore, India, has been announced. He was a member of the Indian Mathematical Society.

Dr. Ferdinand von Lindemann, professor emeritus of mathematics of the University of Munich, died on March 7, 1939, in his eighty-seventh year.

Mr. Albert R. Gallatin, died March 13, 1939, at the age of seventy. He had been a member of the Society since 1927.

Associate Professor W. E. Wilbur of the University of New Hampshire died December 9, 1938.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

152. V. W. Adkisson: *Plane peanian continua with homeomorphisms extendable in the sense of Antoine.*

Let M be a peanian continuum which lies in a plane S, has a finite number of cut points of power greater than 2, and does not separate S. Let M' be any topological map of M in a plane S'. The following conditions are necessary and sufficient that there exist at least one homeomorphism T such that $T(S)=S'$ and $T(M)=M'$: (1) for each cut point P of power 2, at least one of the two components of $M-P$ is symmetric, (2) for each cut point of power greater than 3 the power is finite, and all the components of $M-P$ are homeomorphic with each other except perhaps one. If M is a planar graph, the homeomorphism T exists if and only if M is one of the graphs characterized in the author's thesis, with the one exception noted there (Comptes Rendus des Séances de la Société des Sciences et des Lettres de Varsovie, vol. 23 (1930), pp. 164–193). (Received March 16, 1939.)

153. H. A. Arnold: *Note on completely continuous differentials.*

Given a completely continuous operator $F(x)$ on a Kantorovitch space S to S, suppose $F(x)$ to have a completely continuous differential $F(x_0; dx)$ at x_0. Then if the topological index of the operator $x-F(x_0; x)$ exists, so will the topological index of the transformation $x-F(x)$, and they will be equal. The differential used is one that was first defined by the author, using the idea of approximation to the first difference by means of (abstract) infinitesimals of higher order. The proof of the theorem in question is by means of a metric defined over a single convergent sequence. (Received March 18, 1939.)

154. A. A. Aucoin and W. V. Parker: *Diophantine equations whose members are homogeneous.*

Desboves (Nouvelles Annales de Mathématiques, (2), vol. 18 (1879), p. 481) implies that for $ax^m+by^m=cz^n$ to have a solution in integers, c must always be an integer of a particular kind. If f and g are homogeneous polynomials with integral coefficients, of degrees m and n, respectively, where m and n are relatively prime, then the equation $f(x_1, x_2, \cdots, x_r)=g(y_1, y_2, \cdots, y_s)$ always has solutions in integers. Expressions for these solutions in terms of $r+s$ parameters are obtained. The method of solving is also shown to be applicable to certain nonhomogeneous equations. As an example, solutions are obtained for the equation $ax^2y+bxz+cyz^2=pu^3+qv$, where a, b, c, p, q are integers. (Received March 14, 1939.)

155. Reinhold Baer: *Duality and commutativity of groups.*

Two groups are said to be duals of each other if there exists an anti-isomorphism between their lattices of subgroups. Hamiltonian groups and groups containing elements of infinite order do not possess duals. To be a dual and to be abelian are equivalent properties of finite groups, generated by elements of order a prime number p; and this statement is but one in a larger class of theorems connecting duality and commutativity. If G is an abelian group, then the following three statements are equivalent: (1) there exists a dual to G; (2) the orders of the elements and the orders of the primary components of G are finite; (3) G is self-dual. Finally, fairly large classes of groups are characterized where the existence of a dual implies that the group is "almost abelian." (Received March 7, 1939.)

156. I. A. Barnett and Otto Szász: *On a certain Diophantine equation.*

The authors determine all the solutions of the equation $\cos n\alpha+\cos n\beta=0$ for which $\cos\alpha$ and $\cos\beta$ are rational. It is shown that except for the trivial solutions $\cos\alpha=-\cos\beta$, which exist for all odd n, there are rational solutions if and only if n is contained in one of the two arithmetical sequences $4k-2$ or $6k-3$. Corresponding to each value of k, there are infinitely many rational solutions. However, if $n=4k-2$ where k is of the form $3k'-1$, there are additional trivial solutions. The case $n=4k-2$ leads to the Pythagorean equation $x^2+y^2=1$. Certain generalizations to systems of equations in more than two variables are discussed. (Received March 9, 1939.)

157. H. G. Barone: *Limit points of sequences and their transforms by methods of summability.*

In a paper given before the Mathematical Association of America at Charleston, S.C., March 25, 1939, the following theorem was proved: If $\{s_n\}$ is a bounded complex sequence such that $\lim|s_{n+1}-s_n|=0$, then the set of its limit points is connected. In the present paper, this theorem is applied to sets of limit points of the transforms of bounded complex sequences by the Hölder, Cesàro, Riesz, de la Vallée Poussin, and Euler methods of summability. The set of limit points of the transform of a bounded complex sequence is connected for the Hölder transformation of order r, where r is any positive integer; for the Cesàro transformation of order r, where $Re(r)>0$; for the Riesz transformation of order r, where $Re(r)>0$; for the de la Vallée Poussin transformation; and for the Euler transformation of order r, where $0\leqq r<1$. The set of limit points need not be connected for the Cesàro transformation when $Re(r)\leqq 0$; for the Riesz transformation when r is real and less than or equal to 0; and for the Euler transformation except when r is a real number less than 1. (Received March 8, 1939.)

158. Walter Bartky: *Solution of systems of differential equations by definite integrals.*

A fundamental set of solutions of $dx_i/dt=(A_{ij}+t^{-1}B_{ij})x$, $(i, j=1, \cdots, n)$, with matrix $B_{ij}=a_ib_j$ of rank one, is given by $\int_c e^{\lambda t}(\lambda I-A)^{-1}\phi d\lambda$ where $\phi=I-Be^{-\alpha(\lambda)}\cdot\int e^{\alpha(\lambda)}(\lambda I-A)^{-1}d\lambda$ and $\alpha(\lambda)=b[\log(\lambda I-A)]a$. The contour C has extremities at infinity and loops the characteristic values of A. Similar results are obtainable for $(A_{ij}+tI_{ij})dx_j/dt=B_{ij}x_j$. (Received March 17, 1939.)

159. E. F. Beckenbach and Maxwell Reade: *A characterization of plane isothermic maps.*

Continuing the work reported in abstracts 44-3-92, 44-7-309, and 45-1-76, the authors prove the following theorem: If the functions $x_j(u, v)$, $(j=1, 2, 3)$, are of class C_3 (that is, all derivatives of order less than or equal to three are continuous) in a simply connected domain D, then a necessary and sufficient condition that these functions map D isothermically on a plane surface is that $\sum_{j=1}^{3}[\int_\gamma x_j(u, v)(du+idv)]^2 = 0$ for all closed rectifiable Jordan curves γ lying in D. (Received March 17, 1939.)

160. Clifford Bell: *Solution of numerical equations.*

The usual methods of approximating to an irrational root of a numerical equation are such that the accuracy of a result is not easily obtained, at least by the particular method used alone. However, combinations of methods have been used which will give a measure of the accuracy of any given approximation. In this paper osculatory parabolas are used to obtain approximations, and conditions are stated under which the intercepts of such parabolas bound the irrational root of the given numerical equation. Thus, without the aid of any other devices, the maximum numerical error is known at each step. (Received March 24, 1939.)

161. B. A. Bernstein: *Groups and abelian groups in terms of negative addition.*

The author defines postulationally groups and abelian groups in terms of the "negative" of $a+b$ in (additive) groups and abelian groups. (Received March 16, 1939.)

162. Garrett Birkhoff and John Dyer-Bennet: *A note on real algebraic functions.*

Let $f(x)$ denote any real algebraic function, and let $p(n)$ denote the proportion of the first n terms of the series $f(1), f(2), f(3), \cdots, f(n), \cdots$ which are integers. It is shown that unless $f(x)$ is a polynomial, $p(n)\to 0$ as $n\to\infty$. In other words, the density of integral values assumed by a non-polynomial algebraic function for integral arguments is zero. If $f(x)$ is polynomial of degree d, and has more than d integral values, then it is trivial that its coefficients are rational, and the pattern of its integral values is periodic. Further special results are given. (Received March 31, 1939.)

163. L. M. Blumenthal: *Metric methods in determinant theory.*

This paper is a resumption of the general program, initiated by the writer in earlier articles, of applying the methods of abstract metrics to determinant theory. The behavior of several types of symmetric determinants is studied when certain of their principal minors are subjected to various conditions. Of the chains of new theorems obtained, some were conjectured by the writer in previous papers. One of these chains of theorems (the proof of which is based upon the characterization of pseudospherical sets) is the following: Let $\Delta=|r_{ij}|$, $-1<r_{ij}<1$, $(i\neq j)$, $r_{ii}=1$, $(i, j=1, 2, \cdots, m)$, be a symmetric determinant of order $m>n+3$, where n is any positive integer. If (i) every principal minor of order less than $n+2$ is nonnegative, while every such minor of order $n+2$ vanishes, (ii) at least one principal minor of order $n+3$ does not vanish, then (1) upon multiplying appropriate rows and the same numbered columns of Δ by -1, each element outside the principal diago-

nal has the value $-1/(n+1)$, and (2) each kth order principal minor of Δ equals $(n+2)^{k-1}(n-k+2)/(n+1)^k$, $(1 \leqq k \leqq m)$. Thus the hypotheses (i), (ii) serve to fix (essentially) the value of *every* element of a determinant of type Δ. (Received February 21, 1939.)

164. Richard Brauer: *On groups whose power is divisible by the first order of a prime.* Preliminary report.

Let G be a group of order $g=pg'$ where the prime p does not divide g'. The irreducible representations of G are studied, and a large number of results concerning the group characters of G are obtained. Three applications of this theory may be mentioned. The degrees of the irreducible representations of numerous groups can be calculated. Further, it can be shown that if the commutator group of G has an order divisible by p, then the degrees of the irreducible representations of G are greater than or equal to $(p-1)/2$ (excluding, of course, the 1-representation). This improves, for this class of groups, a result of Blichfeldt. Finally, it follows that a group of order pqr^e, (p, q, r distinct primes), can be simple only if $r=2$, and the difference of p and q is a power of 2. (Received March 10, 1939.)

165. R. H. Cameron and W. T. Martin: *Reciprocals of Fourier-Stieltjes transforms analytic in a strip.*

Conditions are given under which the reciprocal of an analytic Fourier-Stieltjes transform in a strip is itself an analytic Fourier-Stieltjes transform in the strip. The theorm furnishes an extension of the Wiener-Pitt theorem on reciprocals of nonvanishing Fourier-Stieltjes transforms on a line. (Duke Mathematical Journal, vol. 4 (1938), pp. 420–436.) The theorem has applications to integro-difference equations and to difference equations of infinite order. (Received March 10, 1939.)

166. C. C. Camp: *A multiple series expansion.* Preliminary report.

In a former paper the author treated the system $X_j'+[\sum_{i=1}^{p}\lambda_i a_{ji}(x_j)]X_j=0$, $(j=1, 2, \cdots, p)$, with corresponding auxiliary conditions at more than two points in each independent variable. In order to make the denominators in the contour integrals separate after appropriate transformations on the λ's, it was assumed that each $a_{ji}(x_j)$ maintains its average value when integrated over every subinterval involved in the auxiliary condition in x_j. The purpose of the present paper is to remove this restriction. To do this one makes use of an article by H. Poincaré (*Sur les residus des intégrales doubles*, Acta Mathematica, vol. 9 (1886), pp. 321–380) and extends it to the case of p-fold contour integrals for the case of simple poles in each complex variable. With suitable restrictions on the constants of the auxiliary conditions and the extension of certain lemmas from simple to p-fold contours, it is possible to establish the convergence of the appropriate multiple series expansion for the usual type of arbitrary function $f(x_1, x_2, \cdots, x_p)$. (Received March 8, 1939.)

167. H. H. Campaigne: *Product hypergroups.*

The product $A \times B$ of two hypergroups A and B is defined to be the set of all pairs $a \times b$, a in A, b in B. Products are defined thus, $(a' \times b')(a'' \times b'') \supset a \times b$ if and only if $a'a'' \supset a$ and $b'b'' \supset b$. $A \times B$ is a hypergroup. A subproduct $A \circ B$ is a set of such pairs in which each element a of A appears in at least one pair, each element b of B likewise, and $A \circ B$ is a hypergroup. The product as defined is a special case of sub-

product. $A \circ B$ is regular (or semiregular) if and only if both A and B are regular (or semiregular). If A has an idemfactor a_1 such that the set a_1^2 contains only a finite number of elements, then $A \times B$ contains a subhypergroup abstractly identical with A if and only if B contains an idempotent element b_0. An example shows that the theorem cannot be strengthened. If A and B have conjugations among their elements, then there exists a conjugation among the pairs of $A \circ B$ such that $\{A \circ B\} = \{A\} \circ \{B\}$. (Received March 18, 1939.)

168. R. V. Churchill: *On an extension of the Sturm-Liouville development.*

In the problem of determining the temperatures in a slab with one face in contact with a fluid, a homogeneous system of ordinary differential equations arises in which the parameter appears not only in the differential equation but also in one of the boundary conditions. Several other problems in partial differential equations lead to such systems, notably problems in torsional vibrations of shafts with ends subjected to elastic restraints. The temperature problem has been treated quite fully by R. E. Langer in the Tôhoku Mathematical Journal, vol. 35 (1932), pp. 260–275. The characteristic functions do not form an orthogonal set. But by modifications of methods used when the characteristic functions are orthogonal, Langer found the required expansion of the arbitrary initial temperature function and established a theorem of equiconvergence with the Fourier expansion. It is shown in the present paper that by restricting the problem somewhat and requiring greater regularity of the arbitrary function the system can be transformed by a suitable substitution to a standard Sturm-Liouville system, so that the classical theory furnishes the required expansion. (Received March 17, 1939.)

169. L. W. Cohen: *On the mean ergodic theorem.*

It is shown that if T is a linear transformation on a Banach space B to B such that the norms of the iterates T^n of T are bounded, a_{nj} is a regular matrix such that $\sum_{j=k}^{\infty} |a_{n,j+1} - a_{nj}| \to 0$ as $k \to \infty$ uniformly in n, and $L_n x = \sum_{j=1}^{\infty} a_{nj} T^j x$ is a weakly compact set, then there is an $x_0 \in B$ such that $L_n x \to x_0$ and $Tx_0 = x_0$. The matrices of (C, r) with $r > 0$ satisfy the uniformity condition. The case $r = 1$ with Hilbert space as B is the mean ergodic theorem of von Neumann. (Received March 2, 1939.)

170. Harald Cramér: *On the representation of a function by certain Fourier integrals.*

This paper contains necessary and sufficient conditions in order that a complex-valued function $f(t)$ of a real variable be representable as the Fourier-Stieltjes transform of a function $F(t)$ having one of the following properties: (i) $F(x)$ is real, bounded, and never decreasing; (ii) $F(x)$ is of bounded variation in $(-\infty, \infty)$; (iii) $F(x)$ satisfies (ii) and is in addition absolutely continuous. The conditions are partly analogous to those of Hausdorff for a finite interval and are expressed in terms of properties of the Fourier transform of $\mu(\epsilon t) f(t)$, where $\mu(t)$ belongs to a suitably chosen class of kernels. The results extend to functions of several variables, and have applications to the theory of random processes. (Received February 17, 1939.)

171. J. L. Doob: *The law of large numbers for continuous stochastic processes.*

There are certain measurability difficulties which arise when the law of large

numbers is to be applied to a one-parameter family of chance variables. A method of approach is given which overcomes these difficulties, and various forms of the law of large numbers are then proved. The following is an example of the results: Let $\{x_t\}$, $(t \geqq 0)$, be the variables of a centered temporally homogeneous differential process (so that if $0 \leqq t_1 < t_2 < \cdots < t_n$, $x_{t_2} - x_{t_1}, \cdots, x_{t_n} - x_{t_{n-1}}$ are mutually independent), and suppose that the expectation of $x_1 - x_0$ exists: $E(x_1 - x_0) = e$. Then $\lim_{t \to \infty}(x_t - x_0)/t = e$, with probability 1. Certain preliminary results on sums of independent chance variables are needed, of which the following is an example: Let $x_1, x_2, \cdots$ be independent chance variables, and let $\sum_1^\infty x_n = x$ with probability 1. Then if Ex exists, it follows that Ex_n exists for all n, that $Ex = \sum_1^\infty Ex_n$, and that $E(\text{L.U.B.}\ |x_1 + \cdots + x_n|)$ exists. (Received March 16, 1939.)

172. F. G. Dressel: *A class of solutions for the heat equation.* Preliminary report.

The paper shows how to construct for each integer n a solution of the heat equation, whose nth derivative has the properties of the fundamental solution of this equation. The remainder of the paper is concerned with the properties and applications of this set of solutions. (Received March 7, 1939.)

173. D. M. Dribin: *Class field theory of solvable algebraic number fields.*

In the present paper a definition of class field is given which is applicable to non-abelian fields K/k, where k is an algebraic number field. This definition involves a set of prime ideals Π in k, rather than an ideal group. As simple consequences of this definition the analogues of the converse theorem and the uniqueness theorem of the abelian class field theory follow. It is shown that there exist solvable fields K/k which are class fields in this new sense provided that Π be "immersed" in an ideal group H in k. An analogue of the Artin law of reciprocity is also derived. (Received March 10, 1939.)

174. L. A. Dye: *A Cremona transformation associated with the rational normal curve of order n in S_n.*

A projectivity is established between a pencil of quadric primals H and sets of $n-1$ points of the rational normal curve C_n in S_n. The residual intersection of C_n with the prime determined by a general point P and the $n-1$ points of C_n associated with the quadric H passing through P is joined to P by a straight line meeting H again in P'. The points P, P' are conjugate in a transformation of order $6n+3$. The normal curve C_n and three $(n-2)$-dimensional loci of orders 4, 5, and $12n+2$ constitute the fundamental elements of the transformation. The last of these is the locus of parasitic lines. (Received March 1, 1939.)

175. R. A. Favila: *A surface with doubly stratifiable directrices.*

Let S_x be an integral surface of a Fubini system of equations of the form: $x_{uu} = px + \theta_u x_u + \beta x_v$, $x_{vv} = qx + \gamma x_u + \theta_v x_v$, where $\theta = \log \beta\gamma$ and p, q, β, γ are scalar functions of the asymptotic parameters u and v. Consider a pair of Green reciprocal congruences with respect to S_x, and let us denote them by I_1 and I_2. If l_1 and l_2 are any pair of corresponding lines of I_1 and I_2, respectively, then l_1 and l_2 may be taken as the lines joining the pairs of points (Px, Py) and (P_ρ, P_σ), respectively, where $y = x_{uv}$

$-ax_u-bx_v$, $\rho=x_u-bx$, $\sigma=x_v-ax$, and a, b are functions of u and v. Is it possible to determine the surface as well as I_1 and I_2 so that I_1 and I_2 will form a doubly stratifiable couple? The problem has a solution which depends on the integration of an equation of the form $\partial^2\Phi/\partial u\partial v=e^{\Phi}$. The surface is found to have asymptotics belonging to linear complexes, while I_1 and I_2 are directrices of Wilczynski with indeterminate directrix curves. (Received March 15, 1939.)

176. J. M. Feld: *Polygons as fundamental elements in the geometry of plane cubic curves.*

Geometry on a cubic regarded as a locus of a system of polygons was investigated by H. Oppenheimer (Monatshefte für Mathematik and Physik, 1909, p. 141). By defining "collinearity" of polygons, complete sets of polygons, and so on, Oppenheimer showed that theorems known to be true of points on a cubic hold true when certain kinds of polygons are substituted for points. In this paper a new geometry of polygons on a cubic analogous to the geometry of points on a cubic is discussed; the polygons in question are, however, of a type different from those of Oppenheimer. In addition it is shown how any configuration inscribed in a cubic can be used as a basis for the generation of an infinitude of other configurations. (Received March 8, 1939.)

177. A. S. Galbraith and S. E. Warschawski: *On the convergence of Sturm-Liouville series.*

This paper extends in a certain direction some results of M. Krein about the convergence of the derived series of expansions in terms of characteristic solutions of self-adjoint linear differential equations. Let $\{\phi_i(x)\}$ be the characteristic solutions, for values $\{\lambda_i\}$ of the parameter λ_1, of the Sturm-Liouville problem $(py')'+(\lambda r+q)y=0$, $h_1y(a)=h_2y'(a)$, $H_1y(b)=H_2y'(b)$, where p, $r>0$, and p', q, r are continuous. Let n be a positive integer, and if $n\geqq 2$, let $p^{(n-1)}$, $q^{(n-2)}$, $r^{(n-2)}$ be continuous. Then a necessary and sufficient condition that, for an integrable function $f(x)$ with the "Fourier coefficients" $a_i=\int_a^b fr\phi_i dx$, $f^{(n-1)}(x)$ exist and be absolutely continuous in (a, b), and that $\int_a^b[f^{(n)}]^2dx$ exist, is (i) that $\sum_{i=0}^{\infty}a_i^2\lambda_i^n$ converge, (ii) that the functions $G_s(x)$ defined by $G_0\equiv f_j-rG_s=(pG_{s-1}')'+qG_{s-1}$, $(s=1, 2, \cdots, [n/2]-1)$, satisfy the above boundary conditions, and, if n is odd, that $G_{[n/2]}$ vanish at an end point if the boundary conditions require the $\{\phi_i\}$ to do so. As a corollary, it follows that $f^{(m)}(x)\equiv\sum_{i=0}^{\infty}a_i\phi_i^{(m)}(x)$, $(m=0, 1, \cdots, n-1)$, and $\lim_{n\to\infty}\int_a^b[f^{(n)}-\sum_{i=0}^{n}a_i\phi^{(n)}]^2dx=0$. (Received March 11, 1939.)

178. H. H. Goldstine: *Minimum problems in the functional calculus.*

Necessary conditions and sufficient conditions that a functional defined on a region of a Banach space shall be a minimum are determined in terms of the first and second variations of the given functional. Then the more general problem of minimizing a given functional subject to a general side condition is discussed. Under certain hypotheses, necessary conditions for this problem, including a multiplier rule, are obtained; and it is shown that when these conditions are strengthened in the usual manner, sufficient conditions are obtained. The general problem includes both single and multiple integral problems of the calculus of variations as special cases. (Received March 15, 1939.)

179. G. D. Gore: *Transformations for two classes of surfaces.*

This paper develops three transformations for a surface bearing a family of curves

in autoconjugacy of type ν, and for a surface bearing a system of curves in conjugacy of type-ν. The first of these is an analogue of the transformation of Levy for conjugate nets. The second is somewhat analogous to the classical transformation of Laplace for conjugate nets, but has certain characteristics of the transformation of Levy. The third is an analogue of the fundamental transformation F for conjugate nets due to Eisenhart and Jonas. To determine the degree of prevalence of transformations for the above surfaces, a theorem of permutability is established. Earlier transformations for surfaces of these types, due mainly to B. Segre, have involved the linear osculants of order $\nu-1$ and higher to the curves in question, and have involved analytically the derivatives of correspondingly high orders in the point coordinates. The first two of the above transformations involve only the lines tangent to the curves in question, and are therefore accomplished analytically in terms of first derivatives of the point coordinates. (Received March 16, 1939.)

180. T. N. E. Greville: *Invariance of the admissibility of variates under certain general types of transformations.*

This paper develops a set of transformations on variates or "collectives" in terms of which it is possible to formulate completely a large class of problems for which the transformations given by von Mises and other authors are not adequate. These problems arise in the application of the classical theory of probability to the study of frequency distributions of statistical variates. The consistency of the assumptions of the classical theory is investigated by means of the invariance under the transformations of certain properties of the sequences. In an earlier paper the same method was applied to the simple type of collective in which the label space consists of exactly two elements (this Bulletin, abstract 39-7-202). (Received March 8, 1939.)

181. Marshall Hall: *The position of the radical in an algebra.*

The principal result in this paper is that any linear associative algebra is uniquely decomposable as the direct sum of a semi-simple algebra and an algebra bound to its radical. An algebra A is said to be bound to its radical R if when x belongs to A and $xR=Rx=0$ then x belongs to R. Whereas the semi-simple algebra bears no relation to the radical, the bound algebra is largely determined by the radical. In particular, the order of a bound algebra is limited by the order of its radical. For example, if R is of order one, the order of A is at most three and there are only five such algebras in all. (Received March 8, 1939.)

182. O. G. Harrold: *Invariance of the dimensionality of a compact metric space under certain continuous transformations.*

The continuous transformation T carrying A into B is called $(k, 1)$ provided that each point x in B has at most k inverse points. Let T be a $(k, 1)$ transformation defined on the compact metric space A, C the set of all points in B with fewer than k inverses. It is proved in this note that dim. $B=$dim. A if, and only if, dim. $\overline{C}\leqq$dim. A. In particular, if T is exactly $(k, 1)$, dim. $A=$dim. B. (Received March 17, 1939.)

183. Philip Hartman: *On Dirichlet series involving random coefficients.*

This paper deals with the abscissa of convergence C, of uniform convergence U,

of absolute convergence A of "almost all" of the Dirichlet series $\sum \pm a_n/n^s$. It is proved that $1/2 \geqq A-C \geqq U-C \geqq 0$, and that either $1/2=U-C$ or $A-C=0$ is possible. Of considerably more interest is the difference $A-U$; it is shown that $A-U=1/2$ is also possible. The methods employed may be used for a more general class of Dirichlet series. (Received March 7, 1939.)

184. A. E. Heins: *The solution of the discrete equation of heat conduction.*

A solution is derived for the partial difference equation $f(t, y+2c)-2f(t+h, y+c)+f(t, y)=0$, where $f(0, y)$, $f(t, 0)$, and $f(t, a)$ are prescribed. The limiting value of the solution is also considered. (Received March 7, 1939.)

185. M. H. Heins: *Extremal problems for functions analytic and single-valued in a doubly connected region.*

It is well known that certain fundamental function-theoretic inequalities such as Julia's principle of the harmonic majorant, the two constant theorem, Lindelöf's principle, the principle of hyperbolic measure, and so on, are "best possible" when the domain of definition G_z for the functions $w(z)$ involved is simply connected. When one considers, however, functions which are analytic and *single-valued* in a multiply connected region, in general these inequalities are no longer the "best possible," and it is a question of interest to determine effectively the exact bounds in the above inequalities and the extremal functions associated with these bounds. By application of the Poincaré uniformization theorem and the Pick-Nevanlinna theory of interpolation, the exact bounds and their associated extremal functions can be determined effectively when the domain of definition G_z is doubly connected. (Received March 17, 1939.)

186. Olaf Helmer: *A theorem of the Picard type.*

Let ord f and exp f denote the order and exponent of convergence of the integral function $f(z)$. The function $f(z)$ will be called exceptional if exp $f <$ ord f. Let $F(z)$ and $G(z)$ be polynomials, and $A(z)$ and $B(z)$ integral functions with max (ord A, ord B) $<$max (deg F, deg G), then ord $(Ae^F+Be^G)=$max (deg F, deg G). On the basis of this lemma it is possible to prove the following theorem: Let $f(z)$ and $g(z)$ be integral functions of distinct finite orders; then there is at most one integral function $A(z)$ with ord $A <$max (ord f, ord g) for which the function $f(z)+A(z)g(z)$ is exceptional. This is not generally true when $f(z)$ and $g(z)$ are of the same order. We have however the following theorem: If $f(z)$ and $g(z)$ are integral functions of the same finite order ρ, of which at least one is not exceptional, then there are at most two functions $A(z)$ with ord $A<\rho$ for which the function $f(z)+A(z)g(z)$ is exceptional. (Received March 16, 1939.)

187. M. R. Hestenes and H. H. Goldstine: *The condition of Mayer for the problem of Bolza in the calculus of variations.*

Necessary and sufficient conditions that the second variation in the problem of Bolza be positive definite, when the strengthened Clebsch condition holds, have been given by Bliss, Reid, Hestenes, and others. Aside from the condition of Mayer arising from the study of a boundary value problem, no necessary and sufficient conditions for non-negativeness of the second variation have been given. In the present paper,

conditions of this type are established, which when suitably strengthened are necessary and sufficient for the second variation to be positive definite. Conditions of Mayer are given for the fixed end point and the one variable end point problems of Bolza, as well as for the separated end point problem and the generalized problem of Bolza recently studied by Hestenes (abstract 45-1-22). (Received March 16, 1939.)

188. Einar Hille: *Contributions to the theory of Hermitian series.*

A study is made of Hermitian series in the complex plane. Convergent asymptotic expansions are derived for the Hermitian functions $h_n(z)=\exp(-z^2/2)H_n(z)$, leading to a simple determination of the strip of convergence. The differential operator $\delta_z=z^2-d^2/dz^2$ has $h_n(z)$ as characteristic function for the characteristic value $2n+1$. Functions of δ_z are investigated; in particular, entire functions which preserve holomorphism or the property of being an entire function. This leads to a study of singularities of functions defined by Hermitian series and to a gap theorem which is the best of its kind. There exist Hermitian series with a strip of convergence of finite width which represent entire functions. Finally with the given series are associated certain Dirichlet series with exponents $\pm i(2n+1)^{1/2}$ having no other singularities than those of the Hermitian series. (Received March 31, 1939.)

189. P. G. Hoel: *A significance test for minimum rank in factor analysis.*

By treating n variables which are subject to errors of measurement as sums of true and error variables, the true correlation determinant is expressed as a function of n parameters. When all parameters are equal, this determinant reduces to the characteristic polynomial of the original correlation matrix. In factor analysis certain assumptions are made concerning the rank of the true correlation matrix. An inequality is obtained between certain values of the parameters and certain of the characteristic roots which serves as the basis for testing whether or not these assumptions concerning rank are justified. (Received March 3, 1939.)

190. E. V. Huntington: *Stirling's formula with remainder.*

This note gives the Stirling expansion of $n!$, both in the direct and logarithmic forms, through the term involving $1/n^9$, together with a remainder term by which the error involved in stopping the series at any term up to the ninth may be estimated. (Received March 28, 1939.)

191. M. H. Ingraham: *An algorithm for the solution of the unilateral matrix equation.*

An algorithm analogous to that for finding the rational roots of an equation with rational coefficients is given for finding all matrices X with rational elements that satisfy an equation $\sum A_i X^i$ where the A_i are matrices with rational elements. (Received March 16, 1939.)

192. Dunham Jackson: *Orthogonal polynomials on curves of the second degree.*

A theory of polynomials in two real variables orthogonal on the circumference of a circle with respect to an arbitrary weight function is at the same time a theory of

orthogonal polynomials with an arbitrary weight function on an ellipse. But in connection with properties relating to a specific weight function, the transition from circle to ellipse gives rise to problems that are not trivial. By means of a theorem of Peebles on the boundedness of orthogonal trigonometric sums (for the case of polynomials in one real variable see Proceedings of the National Academy of Sciences, vol. 25 (1939), pp. 97–104) it is shown that the ordinary convergence theory of Fourier series carries over to series of orthogonal polynomials on an ellipse with unit weight function and arc length as variable of integration. A corresponding discussion is given for an elliptic arc and for arcs on a hyperbola. (Received March 16, 1939.)

193. Marie M. Johnson: *An extension of a covariant differentiation process.*

In this paper a covariant differentiation process is developed for functions which contain coordinate variables and derivatives of these variables with respect to a parameter. A special case is the process given by H. V. Craig (see this Bulletin, vol. 37 (1931), pp. 731–734). (Received March 16, 1939.)

194. Mark Kac and E. R. van Kampen: *Circular equidistribution and statistical independence.*

In order to obtain a satisfactory basis for the treatment of Buffon's needle problem, the circular equidistribution of an angular variable and a related type of discontinuous distributions are obtained as the only possible solutions of a problem concerning statistical independence. The problem depends on the determination of those transformations $\bar{\phi}_1=\phi_1+f(\phi_2)$, $\bar{\phi}_2=\phi_1$ of a (ϕ_1, ϕ_2)-torus into itself which conserve a given product measure on the torus. (Received January 16, 1939.)

195. Edward Kasner and J. J. DeCicco: *A characterization of the Moebius group of circular transformations.*

The Moebius group of circular transformations is ordinarily defined as the group of point transformations of the plane for which the entire family of ∞^3 circles is invariant. In this paper, the authors wish to find out if it is not sufficient for this definition to require that only some circles shall become circles. If a simple family (∞^3) of circles is understood to be any doubly infinite family possessing the property that there is one and only one circle of the family containing a given lineal element of the plane, then the main result may be stated as follows: If three simple families ($3\,\infty^3$) of circles become circles under a point transformation, then the same is true for all circles, and the point transformation is, therefore, a circular transformation of the Moebius group. For any other point transformation, there are at most two simple families ($2\,\infty^3$) of circles which become circles. The above result gives a minimal characterization of the Moebius group. (Received February 20, 1939.)

196. J. L. Kelley: *A note on the arc-wise connectivity theorem.*

Suppose M is the image of the unit interval I under a continuous transformation T. A simple proof for the existence of an arc between two chosen points x, y of M is obtained by finding a perfect set in I which maps under T into an arc xy. This proof depends only on the properties of compact sets, on the transformation T, and on the fact that the monotone transform of an arc is an arc. (Received March 9, 1939.)

197. R. B. Kershner: *Ergodic curves.*

Let M be an arbitrary bounded plane point set, and let $\epsilon>0$ be fixed. Then a continuous rectifiable curve C is called an ϵ-ergodic curve for M if it is a curve of minimum length having the property that every point of M is at a distance less than or equal to ϵ from C. This paper is devoted to the investigation of such curves (known to exist for arbitrary M and $\epsilon>0$). The main results are that C is simple and has a well defined tangent up to a countable number of corners. (Received March 16, 1939.)

198. Fulton Koehler: *Orthogonal polynomials on certain algebraic curves.*

An investigation is made of systems of orthogonal polynomials in two real variables which correspond to certain contours in the plane of the variables and certain weight functions. The contours considered are specifically a square (see also abstract 43-11-402), a pair of concentric circles, and a pair of intersecting line segments. By relating these systems of polynomials to systems of polynomials in one real variable, sufficient conditions are derived for the boundedness of the normalized polynomials and then for the convergence of series of the polynomials to an arbitrary function. Both uniform convergence and point by point convergence are considered. (Received March 16, 1939.)

199. C. F. Kossack: *The existence of collectives in abstract space.*

This paper considers the existence of admissible sequences or *Kollektivs* in abstract space. Such sequences arise in the study of the foundations of the theory of probability. Beginning with the admissibility conditions given by A. H. Copeland as an interpretation of the von Mises requirements, existence is established by showing that almost every (in the sense of Lebesgue) sequence in abstract space satisfies these conditions. This is first shown with respect to a denumerable field of sets in the space and then for an extended field which is, in general, nondenumerable. The Fubini theorem is then used to prove the existence of sequences satisfying the independence condition for almost every selection of a nondenumerable set. Finally the concept of admissibility is generalized so that the number of conditions has the power of the continuum, and it is shown that a nondenumerable set of sequences satisfies almost all of these generalized conditions. Many of the results previously established by Copeland, Tornier, and Reichenbach are obtained at once from the theorems of this paper merely by specializing the space. (Received March 22, 1939.)

200. C. G. Latimer: *The complete solution of certain Diophantine equations.*

Let f be a classic properly primitive ternary quadratic form, with integral coefficients, whose determinant D is not divisible by a square greater than 1. Let F be the adjoint of f, and let u, v, w be integers, not all zero, such that $f(u, v, w)=0$. There is a one-to-one correspondence between the solutions of $f=0$ and of $F=0$. In this paper the author obtains all solutions, in integers, of $f=0$ and of $F=0$ in terms of u, v, w and certain parameters. The fact that F is the norm of the general element of trace zero in a ring of integral elements in a generalized quaternion algebra is employed. (Received March 17, 1939.)

201. Yuan Lay: *On the imbedding of the skew part into an associative algebra.*

By an algebra is meant here a bilinear vector function. It is studied by splitting it up into symmetric and skew parts. The condition of associativity is translated into relations between these two parts from which, of course, the Jacobi relation for the skew part follows. Two problems arise: what conditions in addition to the Jacobi condition should be imposed on a skew function in order that an associative algebra may exist of which the function is a skew part; and if these conditions are satisfied, how to find the symmetric part. These problems are discussed for certain types of algebras, in particular, for centralized algebras, that is, algebras in which the skew part vanishes only when the arguments are linearly dependent modulo the centrum. A general theorem is proved covering the number of dimensions of the centrum in a centralized algebra, and the problem is solved completely for the cases of three- and four-dimensional space. Some general theorems are given also for certain other cases. (Received March 17, 1939.)

202. A. T. Lonseth: *The problem of Plateau in hyperbolic space.*

Let the hyperbolic geometry of three dimensions be the set $z>0$ of ordinary euclidean space (x, y, z), with Poincaré's metric $ds^2=(dx^2+dy^2+dz^2)/z^2$. Consider surfaces S represented parametrically by equations of the type $x=x(u, v)$, $y=y(u, v)$, $z=z(u, v)$, the coordinate functions being continuous in $u^2+v^2\leq 1$. The area integral is defined, as usual, in terms of the metric, for surfaces whose coordinate functions have continuous first derivatives in $u^2+v^2<1$. It is shown that a Jordan curve, if it bounds a surface of finite (hyperbolic) area, must bound a hyperbolic minimal surface, that is, a surface whose coordinate functions have continuous second derivatives in $u^2+v^2<1$, and satisfy therein the Euler equations which arise from minimizing the area integral. (Received March 18, 1939.)

203. A. N. Lowan: *On some problems in the diffraction of heat.*

The boundary plane $x=0$ of the semi-infinite solid $x>0$, initially at 0°C, is impervious to heat except for an aperture of given shape. If the semi-infinite solid is subjected to an influx of heat at the rate $Q(P, t)$ per unit area, what is the subsequent temperature distribution in the solid? This problem is the thermal equivalent of the problem of diffraction of light. Rigorous solutions are obtained for the following cases: (A) The aperture is an infinite slit between two parallel lines. (B) The aperture is a circle. (C) The aperture is of finite extent and of any arbitrary shape. (D) The aperture consists of the half-plane $x=0$, $y>0$. The Laplace transforms of the temperature function are obtained with the aid of known Fourier or Bessel-Fourier integral representations of functions of one or two variables. The transition from the Laplace transforms to the temperature function is accomplished by means of standard methods in the operational calculus. (Received March 23, 1939.)

204. Saunders MacLane: *The universality of formal power series fields.*

Recently Gleyzal (Proceedings of the National Academy of Sciences, vol. 23 (1937), pp. 581–587) has constructed certain transfinite real numbers and has conjectured that certain corresponding modular fields will be universal; that is, will contain subfields isomorphic to any given field of the same characteristic and the same

cardinal number. It is observed first that the transfinite real numbers are simply the formal power series of Hahn. By using Krull's generalized valuations, the corresponding fields are proved maximal, hence algebraically closed, and thus Gleyzal's conjecture is established. More generally, it is shown that any formal power series field is universal if its coefficient field is algebraically closed and if its group Γ of exponents always contains γ/n when it contains γ. (Received February 24, 1939.)

205. P. T. Maker: *The ergodic theorem for a sequence of functions.*

Let Ω be a space with measure m, $(m(\Omega)<\infty)$, $T(P)$ a measure-preserving transformation of Ω, and $\{f_n(P)\}$ a dominated, convergent sequence of complex-valued functions in $L_1(\Omega)$. For any set of positive integers n_{ij}, $(i, j=1, 2, \cdots; i\leqq j)$, such that $\lim_{i,j\to\infty} n_{ij}=\infty$, $\lim_{j\to\infty} 1/j\sum_{i=0}^{j-1} f_{n_{ij}}(T^iP)$ exists and equals the time mean of $\lim_{n\to\infty} f_n(P)$ except for points P of measure zero independent of the sequence $\{n_{ij}\}$. In the case of the flow, if $f_\lambda(P)$, $(a\leqq\lambda\leqq b)$, is a continuous family of dominated functions of $L_1(\Omega)$, and $\lambda(t, T)$, $(0\leqq t\leqq T\leqq\infty)$, is continuous with range in (a, b), then $\lim_{T\to\infty} 1/T \cdot \int_0^T f_\lambda(t, T)(P_t)dt$ exists for almost all P. (Received March 10, 1939.)

206. H. W. March: *Infinite plane strip of orthotropic material under a concentrated load.*

The deflection of such a strip satisfies the differential equation $D_1\partial^4w/\partial x^4+2K\partial^4w/\partial x^2\partial y^2+D_2\partial^4w/\partial y^4=0$, except at the point of loading (M. T. Huber, Bauingenieur, 1923–1926). After the change of variable $\eta=\epsilon y$ where $\epsilon=(D_1/D_2)^{1/4}$, the differential equation becomes $\partial^4w/\partial x^4+2\kappa\partial^4w/\partial x^2\partial y^2+\partial^4w/\partial y^4=0$ where $\kappa=K/(D_1D_2)^{1/2}$. The transformed differential equation can be used to advantage in treating numerous problems associated with flat plates of orthotropic material. For a strip with simply supported edges, $x=0$ and $x=a$, the deflection is readily expressed in the form $w=\sum Y_n(\eta)\sin n\pi x/a$. When $\kappa<1$, as it is for plywood, the bending moments m_x and m_y are found to be linear combinations of two functions which can be expressed in closed form as the real and imaginary components, respectively, of a function of a complex variable. This form is essentially identical with that given for a function ϕ by A. Nadai (*Elastische Platten*) for a strip of isotropic material, except that a real variable is replaced by a complex variable. (Received March 10, 1939.)

207. W. C. McDaniel: *The deflection of an orthotropic plate.* Preliminary report.

The differential equation for the deflection of an orthotropic plate can be reduced by a linear transformation (H. W. March, abstract 45-5-206) to the form $\partial^4w/\partial x^4+2\kappa\partial^4w/\partial x^2\partial y^2+\partial^4w/\partial y^4=P(x, y)/D_1$. When $\kappa>1$ and $\kappa<1$, the general solutions may be expressed as the sum of arbitrary functions of each of two complex variables. When $\kappa=1$ the equation is the biharmonic equation, and the two complex variables of the other cases reduce to a single complex variable. Series solutions are found for the infinite and semi-infinite strips and the rectangle, with simply supported edges, and having either a concentrated load or a load distributed over a small rectangular area. The solutions for $\kappa=1$ and $\kappa<1$ are found from those for $\kappa>1$ by noting that a quantity $\sigma=(\kappa-1/2)^{1/2}$ becomes zero when $\kappa=1$ and imaginary when $\kappa<1$. The bending moments are given as linear combinations of two functions ϕ and ϕ', which can be expressed in closed form for the infinite and semi-infinite strips with concentrated load. When $\kappa<1$, ϕ and ϕ' become conjugate complex functions, and the bend-

ing moments may be expressed in terms of their real and imaginary parts. (Received March 13, 1939.)

208. E. J. McShane: *On multipliers for Lagrange problems.*

Let $y=y(x)$, $(x_1 \leqq x \leqq x_2)$, minimize $\int f(x, y, y')dx$ in the class of curves satisfying differential equations $\phi_\alpha(x, y, y')=0$, $(\alpha=1, \cdots, m)$, and satisfying certain end conditions. It is well known that it is possible to choose a constant λ_0 and functions $\lambda_\alpha(x)$ in such a way that for the function $F(x, y, y', \lambda) \equiv \lambda_0 f+\lambda_\alpha \phi_\alpha$ the Euler equations and transversality conditions hold. In the present paper it is shown (without reference to normality) that λ_0 and the $\lambda_\alpha(x)$ can be so chosen that the Euler equations, transversality conditions, Weierstrass condition, and Clebsch condition are all satisfied. (Received March 10, 1939.)

209. E. J. McShane: *The condition of Legendre for double integral problems of the calculus of variations.*

Let $x=x(u, v)$ (that is, $x^i=x^i(u, v)$, $i=1, \cdots, n$), minimize the integral $\iint f(u, v, x, p, q)dudv$, where p, q are the partial derivatives of x with respect to u, v respectively. The analogue of the Legendre condition $\xi^\alpha f_{p^\alpha p^\beta} \xi^\beta+2\xi^\alpha f_{p^\alpha q^\beta} \eta^\beta+\eta^\alpha f_{q^\alpha q^\beta} \eta^\beta \geqq 0$ is known to hold if ξ and η are linearly dependent. Otherwise it may fail. It has been shown by Albert and by Reid that if $n=2$ the condition (1) $\xi^\alpha F_{p^\alpha p^\beta} \xi^\beta+2\xi^\alpha F_{p^\alpha q^\beta} \eta^\beta+\eta^\alpha F_{q^\alpha q^\beta} \eta^\beta \geqq 0$ holds for all ξ and η, the function F being the sum of f and a multiple of the jacobian of x^1, x^2 with respect to u, v. Here it is shown that if $n=3$ inequality (1) holds, F being the sum of f and a linear combination of the three jacobians of the x^i with respect to u, v. (Received March 10, 1939.)

210. A. D. Michal and A. B. Mewborn: *General projective differential geometry of paths.*

In this paper the authors study the invariant properties of the differential equation of paths in a Hausdorff topological space with a Banach coordinate space B, without postulated inner product, under projective changes of connection (see A. D. Michal, Proceedings of the National Academy of Sciences, vol. 23, p. 547, equation 4). A new element of structure is introduced by defining a geometric object with components which we call the *gauge form* Γ^0. This, in turn, is used to define distinguished parameters, called *projective normal parameters*, and a projective connection Π with components in a composite Banach coordinate space B_1 of couples (x, x^0), where x is in B and x^0, called the *gauge variable*, is real. This connection Π is a linear connection under a certain restricted set of coordinate transformations in B_1. By a proper choice of the coordinate space B, several new infinite dimensional cases are obtained as well as important finite dimensional cases from a somewhat different viewpoint. (Received March 17, 1939.)

211. Harlan C. Miller: *A separation theorem.*

The following result has been obtained: If M is a compact continuous curve, A and B are two points of M, and G is an uncountable collection of mutually exclusive point sets such that each set of G consists of two points and irreducibly separates A from B in M, then there exists an uncountable collection H of mutually exclusive sets such that each set of H contains two points and irreducibly separates A from B in M and no set of H separates two points of another set of H from each other in M, and M contains two arcs XPY and XQY having only X and Y in common such that

one point of each set of H belongs to XPY and the other to XQY, and if K denotes the sum of all the sets of H, then $K \cdot XPY$ and $K \cdot XQY$ are closed. (Received March 20, 1939.)

212. Harlan C. Miller: *On the characterization of a certain type of continuous curve.*

The following results have been obtained: If every two points of a locally compact continuum M are irreducibly separated from each other in M by some pair of points, then for every two points A and B of M there exist uncountably many mutually exclusive pairs of points each irreducibly separating A from B in M. A necessary and sufficient condition that every two points of a locally compact continuum M be irreducibly separated from each other in M by some pair of points is that M be a continuous curve which contains no two simple closed curves having more than one point in common and which also has the property that if A and B are two of its points which are separated from each other in M by some point, the set of all points which separate A from B in M is totally disconnected. (Received March 20, 1939.)

213. M. G. Moore: *On fundamental systems of solutions for linear difference equations.*

Consider the difference equation $\sum_{\mu=1}^{N} c_\mu f(x+\alpha_\mu)=0$, where $N \geqq 2$ and α_μ and c_μ are arbitrary complex constants such that $c_\mu \neq 0$, $\alpha_\mu \neq \alpha_\nu$ for $\mu \neq \nu$. If σ is a zero of the generating function $h(t)=\sum_{\mu=1}^{N} c_\mu \exp(\alpha_\mu t)$, the difference equation is satisfied by $\exp(\sigma x)$, and, if ω is the multiplicity of σ, is also satisfied by $x^q \exp(\sigma x)$, $(q=1, 2, \cdots, \omega-1)$. These solutions are called fundamental solutions. On certain sets of curves in the complex plane, it is shown that all Lebesgue integrable solutions of the difference equation may be approximated as closely as desired by linear combinations of fundamental solutions. All solutions analytic in the neighborhoods of the points α_μ may be approximated uniformly in certain regions of the plane. The principal tool used is the property of biorthogonality discovered by the author (see abstract 44-1-41). (Received March 17, 1939.)

214. C. B. Morrey: *Multiple Denjoy-Perron integrals.*

In this paper, the special and general Denjoy-Perron integrals are generalized to n dimensions. The Perron form of the definition is used first as this exhibits most clearly the naturalness of the generalization. All the usual theorems for the Denjoy integrals are proved and it is shown that the multiple integrals reduce to the ordinary Denjoy integrals in one dimension. Fubini's theorem is proved, and it is shown that if $f(x)$ and $\phi(y)$ are both integrable D (or D^*) in m and n variables, respectively, then $f(x) \cdot g(y)$ is integrable D (or D^*) in $m+n$ variables. The integral includes the Lebesgue integral. Equivalent descriptive and constructive definitions are developed. Neither of the above integrals is equivalent to any previously defined multiple integral. An example is given of a continuous function $f(x, y)$ which possesses a partial derivative $\partial f/\partial x$ at each point which, however, is not two dimensionally integrable. (Received March 17, 1939.)

215. Marston Morse and C. B. Tompkins: *The existence of minimal surfaces of general critical type.* II.

The authors begin with a proof that the area of a harmonic surface of the type of a disc bounded by a simple rectifiable curve varies continuously with the curve,

convergence in the space of the curves being convergence in variation. They then show how the hypotheses on the curve g, in the paper I with the same title in the Annals of Mathematics, can be weakened by the method of approximation, using the results of I. In the special case of the theorem that two disconnected minimizing critical sets imply a non-minimizing minimal surface it is sufficient to assume that g is rectifiable and that the ratio of chord to corresponding minimum arc length on g is bounded from zero. More generally, Theorem 7.4 of I can be established if g is rectifiable and can be approximated by curves g_n admissible in I, and such that the ratio of the lengths of corresponding chords on g_n and on g tends uniformly to 1 as n becomes infinite and the length of the chord on g tends to zero. These conditions are considerably weaker than the condition that the curve be of class C'. (Received March 11, 1939.)

216. H. T. Muhly: *Valuations and infinitely near algebraic loci.* Preliminary report.

In the theory of continuous systems of surfaces in S_3, base conditions imposed by infinitely near base curves play an important role. These conditions can be described by the valuation ideals (v-ideals) in the ring $K(z)[x, y]=\Delta[x, y]$, belonging to zero dimensional valuations B of $\Sigma=\Delta(x, y)$. The author investigates these valuations of Σ for an arbitrary coefficient field Δ. Use is made of the results obtained by O. Zariski in the case where Δ is algebraically closed. It is proved that on extending Δ to its algebraic closure $\bar{\Delta}$, any $B(\Sigma)$ decomposes into a set $\bar{B}_1, \cdots, \bar{B}_n$ of valuations of $\bar{\Delta}(x, y)$ which are conjugate over Δ. If $\{Q_\alpha\}$ is the B-sequence of simple v-ideals in $\Delta[x, y]$ and $\{\mathfrak{Q}_{\alpha i}\}$ is the B_i-sequence in $\Delta[x, y]=\mathfrak{O}$, the main lemma states that $\mathfrak{O}Q_\alpha=\mathfrak{Q}_{\alpha i_1}\cdot\mathfrak{Q}_{\alpha i_2}\cdot\cdots\cdot\mathfrak{Q}_{\alpha i_s}$ where $\mathfrak{Q}_{\alpha i_1}, \cdots, \mathfrak{Q}_{\alpha i_s}$ are the distinct ideals among $\mathfrak{Q}_{\alpha 1}, \cdots, \mathfrak{Q}_{\alpha n}$. This lemma enables one to relate the properties of B to those of $\bar{B}_i$. The lemma yields first the unique factorization theorem for products of simple ideals (complete ideals) in $\Delta[x, y]$, and second that the Q_β, $(\beta<\alpha)$, are uniquely determined by Q_α, that is, the Q_β are independent of the valuation to which Q_α belongs. These results serve as a starting point for a theory of infinitely near curves in S_3 or, more generally, infinitely near V_{n-1} in S_n. (Received March 8, 1939.)

217. I. M. Niven: *On a certain partition function.*

Consider the function $F(x)=f(x)f(x^4)f^{-1}(x^2)f^{-1}(x^3)$, where $f(x)=\prod_{i=1}(1-x^i)^{-1}$. It is a power series in x, convergent within the unit circle, the coefficient a_m of x^m being the number of partitions of an integer m, the summands being of the form $6n\pm1$. Schur (Sitzungsberichte der Akademie der Wissenschaften, 1926, pp. 488–495) has show that the number a_m of partitions of m with summands of the form $6n\pm1$ equals the number of partitions of m such that the difference between any two summands is at least three, and at least six in case both summands are divisible by three. By using the Rademacher modification of the Hardy-Ramanujan method, and Kloosterman sums of roots of unity, the values of the a_m are obtained in the form of convergent series involving Bessel functions. (Received February 24, 1939.)

218. Rufus Oldenburger: *Completely reducible forms.*

For any field F the minimal number of a completely reducible form (that is, a form which splits into linear factors) of degree n is not greater than 2^{n-1}. From this, the theory of nonsingular cubic forms introduced in a paper by the author in the Transactions of this Society, and minimal representations of such forms, the author

solves the problem of complete reducibility of cubic forms for any field F with characteristic different from 2, 3. (Received March 9, 1939.)

219. G. H. Peebles: *The boundedness of certain systems of orthogonal functions.*

The purpose of this paper is to reformulate and extend certain results on the boundedness of orthonormal polynomials obtained by the writer in another paper (Proceedings of the National Academy of Sciences, vol. 25 (1939), pp. 97–104). The method, which does not depend on asymptotic formulas, is sufficiently simple and general to be of interest even in the case of Jacobi polynomials and other orthogonal polynomials for which the results are known. As applied to orthogonal trigonometric sums, it gives new results, which are then susceptible of further application in the corresponding convergence theory. (Received March 16, 1939.)

220. R. S. Phillips: *Integration in a convex linear topological space.*

This paper is essentially an extension of Birkhoff's generalization of the Lebesgue integral to a convex linear topological space. The chief difference lies in the replacement of unconditionally convergent sequences of sets by unconditionally summable sequences of sets for which it is required that the terms remain within a given neighborhood for sufficiently large terms and this independent of order. The integral is an absolutely continuous and completely additive set function depending linearly on the argument. Convergence theorems, the integrability of the product of an integrable function and a bounded summable function, and differentiation are discussed. Finally it is shown that the recently defined integrals of Birkhoff, Dunford, Gelfand, and Pettis are special instances of this integral. (Received March 16, 1939.)

221. R. S. Phillips: *On additive set functions.*

This paper is concerned with additive functions on a sigma-family of sets to real numbers or to a Banach space. The first section establishes for both of these cases a decomposition theorem in which the function is expressed as a denumerable sum. To each of the component functions corresponds a different cardinal number such that it is non-trivially defined on a set of this power and vanishes for all sets of lower power. It is shown that each additive family of sets which constitutes a separable space when metrized by a measure function is in a sense equivalent to the Lebesgue measurable sets on (0, 1). Then a non-separable set space is defined in a nondenumerable torus space. These results are applied to integration in vector spaces. An example is given of a function Pettis integrable but not Birkhoff integrable and another which is Dunford integrable but not Pettis integrable. Finally differentiation of functions defined to a Banach space is considered. (Received March 16, 1939.)

222. W. T. Puckett: *The images of 2-dimensional surfaces under 0-regular transformations.*

The transformation $T(A)=B$ is said to be 0-regular (see A. D. Wallace, abstract 44-3-161) if for every sequence of points $\{y_i\}$ converging to y in B the sequence $\{T^{-1}(y_i)\}$ converges 0-regularly to $T^{-1}(y)$. If A is a 2-dimensional pseudo-manifold and T is monotone 0-regular, either T is a homeomorphism or B is an arc or a simple closed curve. Moreover, if T is any 0-regular transformation, either B is a pseudo-manifold, an arc, or a simple closed curve. (Received March 9, 1939.)

223. Tibor Radó: *On cyclic elements.*

The fundamental concept of the cyclic element theory, as developed by Kuratowski and G. T. Whyburn (Fundamenta Mathematicae, vol. 16, pp. 305–331) is the concept of pairs of conjugate points of a Peano space S. Two points a, b of S are *conjugate* if for every choice of a point p, different from a and b, the points a and b are located in the same component of $S-p$. Write $a\sim b$ to express this fact. The binary relation $a\sim b$ is clearly determinative, symmetric, and reflexive, but generally it is *not* transitive. That is, $a\sim b\sim c$ does not imply, generally, that $a\sim c$. The present paper is based on the remark, apparently unnoticed so far, that if there exist two distinct points b_1, b_2 such that $a\sim b_1\sim c$, $a\sim b_2\sim c$, then it *does* follow that $a\sim c$. The purpose of the paper is to present a treatment of the cyclic element theory based on this *weak transitivity* property of the binary relation $a\sim b$. (Received March 6, 1939.)

224. Tibor Radó and P. V. Reichelderfer: *Some properties of continuous transformations in the plane.*

Given, on a fundamental rectangle in the w-plane, a continuous transformation carrying this rectangle into a flat surface in the z-plane, Radó (Duke Mathematical Journal, vol. 4 (1938), pp. 189–221) defines a multiplicity for each point of the z-plane, the kernel or essential multiplicity for the surface, this multiplicity being defined in terms of transformations close to the given one. In the present paper an intrinsic characterization of this multiplicity is developed. Given any point in the z-plane, this multiplicity is defined with reference to the image under the given transformation of every rectangle in the fundamental rectangle; thus a function of rectangles arises. Conditions for the existence of a completely additive extension of this function of rectangles, together with several closely related functions of rectangles, to a class including all Borel sets in the fundamental rectangle are derived, and the extent to which such extensions are unique is discussed. (Received March 10, 1939.)

225. Tibor Radó and J. W. T. Youngs: *On upper semicontinuous collections.*

During the course of the years 1924 to 1926, R. L. Moore, P. Alexandroff, and B. de Kerékjártó considered certain collections of sets which were called upper semicontinuous by Moore. Apart from the fundamental ideas, however, their work does not overlap: the space, the terminology, and the application are different in each case. The obvious importance of their work, together with the wide range of the applications, makes it interesting to observe that their definitions, though worded in entirely different fashions, become equivalent if applied in a certain conveniently chosen abstract space. This equivalence is rather obvious, but the purpose of this paper is to establish it in a way which is not suggested immediately by their work. Indeed, it will be shown that the definitions proposed represent in a sense the unique answer to a certain simple and fundamental question, and hence are necessarily equivalent. (Received March 9, 1939.)

226. E. D. Rainville: *Linear operational equations and the Laplace transformation.*

Some of the results of a previous paper (abstract 44-11-455) are applied to the solution of operational equations in σ (essentially the Laplace integral transforma-

tion) with coefficients which are linear differential operators. The methods employed are considered in the light of a more general theory of certain operators of any class and of finite order. Some of the results are carried over to the less restricted operators. Others, particularly because of the existence of a fundamental system of two linear differential operators invariant under σ, are peculiar to the operator σ. (Received March 14, 1939.)

227. E. D. Rainville: *Linear partial differential operators and the Laplace transformation.*

In this paper some of the results of a previous paper (abstract 44-11-455) are extended. The study of iterated σ transformations is brought to the stage where linear partial operational equations in σ may be solved. The methods used center around an expansion theorem for linear partial differential operators in direct analogy to one of the methods used in the case of one independent variable. (Received March 14, 1939.)

228. E. D. Rainville: *Relations among certain operators of class three.*

The paper considers linear operators which are defined as acting on ordinary linear differential operators. Relations are obtained between certain of these operators including, in particular, the σ and ψ of a previous paper (abstract 44-11-455) and the classical adjoint operator. (Received March 14, 1939.)

229. W. C. Randels: *On the absolute summability of Fourier series.* II.

Bosanquet (Proceedings of the London Mathematical Society, vol. 41, p. 517) gives conditions for absolute summability $C(\alpha)$ which imply that $C(\alpha)$ is a local property for $\alpha>1$, but the question for $\alpha\leqq 1$ is not answered. In this paper it is shown by an example that it is not a local property for $\alpha=1$. (Received March 3, 1939.)

230. W. T. Reid: *A note on the Du Bois-Reymond equations in the calculus of variations.*

McShane (Mathematische Annalen, vol. 109 (1934), pp. 746–755) and Tonelli (Annali della R. Scuola Normale Superiore di Pisa, vol. 3 (1934), pp. 213–237; pp. 214–219, in particular) have given conditions on the integrand function of a simple integral problem of the calculus of variations which insure that every absolutely continuous minimizing arc satisfies the Du Bois-Reymond form of the Euler equations. The principal purpose of the present note is to give a very simple proof of these results of McShane and Tonelli. Basically, the proof here given is intimately related to the proof of the fundamental lemma as given by Bliss (*Calculus of Variations*, Open Court, 1925, pp. 20–21). (Received March 17, 1939.)

231. J. H. Roberts: *A note on topological mappings.*

Examples have been given of acyclic curves having one of the following properties: (1) If f is a topological mapping of M into a subset of M, then $f(M)=M$. (2) If f is a topological mapping of M such that $f(M)=M$, then for each $p \in M$, $f(p)=p$. In the present note an acyclic continuous curve M is defined which has *both* of these prop-

erties; that is, the only topological mapping of M into a subset (proper or improper) of M is the identity. (Received March 8, 1939.)

232. M. S. Robertson: *The variation of the sign of v for an analytic function u+iv.*

Let $f(z)=u+iv\not\equiv 0$ be regular within the unit circle. If z traces out the circle $|z|=r<1$ once, starting at any point on the circle at which $f(z)$ does not vanish, during the circuit either v is of constant sign or it changes sign an even number of times. Let p be a fixed nonnegative integer, and suppose that, on each circle $|z|=r<1$, v does not change sign more than $2p$ times. The author obtains a representation of the form $f(z)=\phi^p(z)\cdot F(z)$, $\phi(0)=0$, $\phi'(0)=1$, where $\phi(z)$ is star-like with respect to the unit circle, and where, for $|z|<1$, $F(z)$ is the limit of functions $F_n(z)$ which are regular for $0<|z|\leqq 1$ and which have the property that on $|z|=1$ the real part of $F_n(z)$ is nonnegative. At the origin each $F_n(z)$, when not regular there, has a pole of order not exceeding p. From this representation a study is made of the rate of growth of $f(z)$ as $|z|\to 1$. Inequalities are obtained for the coefficients of the power series of $f(z)$. (Received March 9, 1939.)

233. R. M. Robinson: *On numerical bounds in Schottky's theorem.*

If the function $f(z)$ is regular and different from ± 1 for $|z|<1$, and if $f(0)=a$, then, according to Schottky's theorem, $|f(z)|$ is bounded for $|z|\leqq r$ by a number depending only on a and r. Let $K(a, r)$ be the smallest such number. It is shown that $8K(a, r)-10<(8|a|+10)^{(1+r)/(1-r)}$, and that $8K(a, r)+10>(8|a|-10)^{(1+r)/(1-r)}$ provided $8|a|-10>0$. Therefore one has obtained (for the first time) numerical bounds for $K(a, r)$, which are close enough so that the asymptotic formula $8K(a, r)\sim(8|a|)^{(1+r)/(1-r)}$, for $a\to\infty$ and fixed r, will follow from them. The proof uses the geometrical properties of the modular function, defined as the function which maps a circular-arc triangle, with zero angles, on a half-plane. (Received March 10, 1939.)

234. O. F. G. Schilling: *Units in p-adic algebras.*

Let A be a simple algebra over the field of p-adic numbers. The author investigates the structure of unit groups in orders of A. It turns out that every unit group has a finite base so that each unit can be represented as an infinite convergent product. Convergence is to be understood in terms of the unique pseudo-valuation of the given algebra. (Received March 2, 1939.)

235. H. M. Schwartz: *A study of a certain class of continued fractions.*

This paper is an extension of the author's abstract 44-5-262. The continued fraction $k_1/(z-c_1)+k_2/(z-c_2)+\cdots$ (k_i and c_i complex, $k_i\neq 0$, z a complex variable) is studied under certain restrictions on the coefficients which lead to a number of cases in which the numerators and denominators of the approximants, when multiplied by suitable factors, converge uniformly in certain regions of the z-plane. The nature of the limit functions is discussed. As an application, the continued fractions representing the quotient $J_\nu(z)/J_{\nu-1}(z)$ ($J\nu(z)$—Bessel's function) and the related system of Lommel polynomials are studied. The latter are shown to form an orthogonal set for all values of the parameter ν, real or complex, under an extended definition of orthogo-

nality. Application is also made to a certain special case in the moments problem of Stieltjes and Hamburger. (Received March 9, 1939.)

236. G. E. Schweigert: *A note on the limit of orbits.*

This note sets forth a proposition based on a paper published recently (Duke Mathematical Journal, vol. 4 (1938), pp. 719–724) by D. W. Hall and the author. Let $T(M)=M$ be a pointwise periodic transformation of a compact metric space onto itself. Let L be the limit set for a convergent sequence of orbits each of which consists of a point x together with its images. Under these conditions, it is found that L contains only a finite number of components which permute under the transformation. An example is given to show the effect of this theorem in the case of a well known dendrite. (Received March 15, 1939.)

237. W. T. Scott and H. S. Wall: *Power series in which each exponent is at least twice the preceding.*

In the corresponding continued fraction $1+a_1x^{\alpha_1}/1+a_2x^{\alpha_2}/1+a_3x^{\alpha_3}/1+\cdots$ for the power series $1+\sum_{i=0}^{\infty}c_{i+1}x^{\lambda_i}$, ($c_i\neq 0$, $\lambda_i\geqq 2\lambda_{i-1}$, $i=0, 1, 2, 3, \cdots$), the a_n are independent of the λ_n and the α_n are independent of the c_n, and simple formulas exist for the a_n, α_n in terms of the power series and for the c_n, λ_n in terms of the continued fraction. For example, $\lambda_n=\alpha_1+\alpha_2+\alpha_5+\cdots+\alpha_k$, ($k=2^{n+1}-1$, $n=0, 1, 2, 3, \cdots$). The corresponding continued fractions for the derivative, and for the reciprocal, of the power series are easily obtained from that for the power series itself. If $1+\sum_{i=0}^{\infty}d_{i+1}x^{\mu_i}$ is another series of the same type, then the corresponding continued fraction for the "composite" series $1+\sum_{i=0}^{\infty}c_{i+1}d_{i+1}x^{\lambda_i+\mu_i}$ is obtained from the separate corresponding continued fractions by the same sort of composition except for possible changes of signs in the partial numerators. The convergents of the continued fraction are Padé approximants for the series and fill all the "fields" (i, j) of the Padé table for which $j\geqq i$. (Received March 13, 1939.)

238. D. M. Seward: *Note on an integral equation.*

The author proves the following theorem. Let k be a positive constant. Let $F(x)$ $x=x_1+ix_2$, be analytic and bounded in the domain D: $|x_2|<k(1-x_1)$, $|x_2|<k(1+x_1)$ Let $K_1(x, y)$ be analytic in the domain D_1: x in D, y in D, $y_1<x_1-|x_2-y_2|/k$. Let $K_2(x, y)$ be analytic in the domain D_2: x in D, y in D, $y_1>x_1+|x_2-y_2|/k$. Let both K_1 and K_2 satisfy $|K_i(x, y)|<M|x-y|^{-\alpha}$ where $\alpha<1$ and M are constants. Let λ be a non-characteristic value for the equation $f(x)=F(x)+\lambda\int_{-1}^{x}K_1(x, y)f(y)dy$ $+\lambda\int_{x}^{1}K_2(x, y)f(y)dy$, x and y real. Then this equation has a unique solution $f(x)$, analytic and bounded in D. (Received March 10, 1939.)

239. J. A. Shohat: *On the Lagrange interpolation formula based on the zeros of orthogonal Tchebycheff polynomials.*

The author considers the Lagrange interpolation formula based on the zeros of orthogonal Tchebycheff polynomials (OP) corresponding to any given distribution function. A horizontal step-function is introduced which has the first $2n$ moments of the given distribution. The interpolation polynomial is now represented as a finite expansion in series of OP corresponding to the above step-function. Various properties of this polynomial are given, and applications made to polynomials in general. (Received March 9, 1939.)

240. J. A. Shohat and Vivian E. Spencer: *Mechanical quadratures coefficients as functions of the moments.*

Extending a method of Markoff for the zeros of orthogonal polynomials (*OP*) theorems have been developed concerning the behavior of the coefficients of the associated mechanical quadratures formula as functions of the moments. Applications are made to various special classes of *OP*. (Received March 9, 1939.)

241. P. M. Swingle: *A finitely containing connected set.*

In this paper, using the hypothesis of the continuum and a modification of the method used by E. W. Miller (Fundamenta Mathematicae, vol. 29, pp. 123–133), it is shown that there exists a connected set which, for every positive integer n greater than one, is the sum of n mutually exclusive biconnected subsets but does not contain infinitely many mutually exclusive connected subsets. This solves a number of problems previously proposed (American Journal of Mathematics, vol. 53 (1931), pp. 394–395). (Received March 18, 1939.)

242. Otto Szász: *On the order of the partial sums of Fourier power series.*

It is shown by a simple construction that corresponding to any sequence $d_n \to 0$, there exist Fourier power series $\sum c_n e^{inz} \sim F(e^{iz})$ such that $\limsup_{n\to\infty} \sum_0^n c_\nu / n d_n = +\infty$. Moreover, a slight change in the coefficients provides a function $F(z)$ having the same property and such that $F(z)/(1-z)$ is a generalized Fourier power series for $|z| = 1$. (Received March 9, 1939.)

243. R. M. Thrall: *Trilinear forms and determinental manifolds.* Preliminary report.

Let $(x_{\alpha\beta})$ be a p by q matrix whose elements are linear forms in $x_1, \cdots, x_n$, say $x_{\alpha\beta} = a_{\alpha\beta\gamma} x_\gamma$, and denote by V_r the manifold defined by the vanishing of the $(r+1)$-rowed minors of $(x_{\alpha\beta})$. Then the projective invariants of V will be invariants of the trilinear form whose coefficients are $a_{\alpha\beta\gamma}$. The purpose of this paper is the application to trilinear form classification of the theory of the manifolds V as developed by T. G. Room, *The Geometry of Determinental Loci*, Cambridge University Press, 1938. In the general case (that is, coefficients arbitrary and excluding certain values of p, q, n), it is possible to give a complete set of invariants for the trilinear form in terms of its related manifolds V. The significance of special cases is also discussed. (Received March 14, 1939.)

244. H. E. Vaughan: *Locally peripherally compact spaces.*

It has been shown by Zippin (this Bulletin, vol. 40 (1934), p. 56) that any complete separable metric space having the property that every point has arbitrarily small neighborhoods with compact boundaries (local peripheral compactness) may be compactified by the addition of a denumerable set of points. The object of the present paper is to show that a space having the above properties with the exception of completeness may be imbedded in a complete locally peripherally compact space which differs from it by a zero-dimensional set; and, hence, may be imbedded in a compact metric space which differs from it by the sum of a zero-dimensional set and a denumerable set. If a determining system for the original space can be chosen which consists of neighborhoods with compact boundaries all of which are of dimension at most

$n-1$, the imbedding space will also be at most n-dimensional. (Received March 17, 1939.)

245. C. W. Vickery: *Deformations of frequency functions.*

If $P(x_1, \cdots, x_k)$ and $Q(x_1, \cdots, x_k)$ are frequency functions, $\beta(x_1, \cdots, x_k) = Q(x_1, \cdots, x_k)/P(x_1, \cdots, x_k)$ is a *deformation* of P; a *continuous deformation* provided β is a continuous function. The identity deformation is produced when $\beta \equiv 1$. The mean value of β with respect to P is 1. Every continuous deformation leaves at least one point with fixed probability. The second moment of β about the identity deformation is a measure of deformation. Under certain conditions there exists a transformation of variables equivalent to deformation β and β is equal to the reciprocal of the absolute value of the jacobian of the transformation. If $Q(x)$ and $\beta(x)$ are estimated from a sample, $\beta(x)$ may be supposed an algebraic function of degree n. As an estimate of $\beta(x)$ one may use $\beta'(x)$, that algebraic deformation of degree n which minimizes χ^2 for the sample distribution with respect to $P(x) \cdot \beta'(x)$. If $\beta'(x)$ is linear, its slope is a measure of linear bias. One may thus classify and measure the bias of samples and test the hypothesis that a sample S is drawn with a particular form of bias from a population π. (Received March 29, 1939.)

246. C. W. Vickery: *Random and biased sampling machines.*

A machine has been designed for drawing independent samples at random or, with a predetermined form of bias, from a population of given distribution. One form of this machine consists essentially of a Rutherford-Geiger counter provided with a source of α rays and attached through an amplifier to a device for drawing cards. As cards flow through the machine, each α particle actuates the drawing of a card. The ratio of the rate of detected emission s to the rate of flow of the cards r is adjusted to produce the desired size of sample. The rate s is varied by varying the distance of the radioactive substance from the detecting vessel, thus introducing various forms of bias. A varying r equivalent to a varying s produces a transformation equivalent to a deformation. Another form of the machine dispenses with cards and records the emissions on a chronograph tape. The constant speed of a tape with linear scale produces a sample from a rectangular distribution. The scale or speed of the tape may be varied to obtain samples from other distributions, or a transformation equivalent to a deformation. (Received March 29, 1939.)

247. R. K. Wakerling: *On the loci of $(k+1)$-secant k-spaces of a curve in r-space.*

In r-space the $(k+1)$-secant k-spaces of a general curve form a $(2k+1)$-dimensional variety V, $(2k+1<r)$. If the curve is rational, the properties of V, and of its sections by spaces of lower dimension, can be studied by means of a (1, 1) correspondence which may be established between the k-spaces of V and the points of a $(k+1)$-space. This correspondence is constructed by employing for the space of representation a fixed, osculating $(k+1)$-space having $(k+1)$-point contact with the curve. It is found that there is a close connection between the correspondence in question and the Cremona $(r-k)$-*ic* transformation in an S_{r-k} of S_r. (Received March 14, 1939.)

248. A. D. Wallace: *On ergodic continua.*

Using the linear measure of continua as developed by Frink (American Journal

of Mathematics, vol. 58 (1936), p. 514) the results of Martin on ergodic curves (American Journal of Mathematics, vol. 58 (1936), p. 727) are extended to continua in a compact metric space. It is shown that certain of Martin's theorems are consequences of theorems on lower semicontinuous functions. In particular, it is shown that the ergodic function is monotone nonincreasing and continuous on the right (Martin, Theorem 3). (Received March 8, 1939.)

249. A. D. Wallace: *Some cyclicly extensible and reducible properties.*

It is assumed that S is a Peano continuum and that P is a property of point sets. The following properties are considered. $\Delta_0(P)$: If S is the sum of two continua, their product has property P. $\Delta_1(P)$: The boundary of each component of the complement of a continuum has property P. $\Delta_2(P)$: Every irreducible separation of S between two points has property P. $\Delta_3(P)$: If X and Y are disjoint closed sets containing x and y, there exists a set Z having property P which separates S between x and y and is disjoint with $X+Y$. It is known that for any property P each of the above properties implies the following. In this paper the following theorems are proved: (1) For any property P, $\Delta_0(P)$, $\Delta_1(P)$, and $\Delta_2(P)$ are cyclicly reducible. (2) If P is such that X having P implies that XE has P for each true cyclic element E of S, then $\Delta_3(P)$ is cyclicly reducible. (3) If each point has property P, then $\Delta_2(P)$ and $\Delta_3(P)$ are cyclicly extensible. From these results several known theorems are obtained. (Received March 8, 1939.)

250. J. L. Walsh: *On interpolation by functions analytic and bounded in a given region.*

Let C be a Jordan curve interior to a second Jordan curve B. Let the points $\beta_{n1}, \beta_{n2}, \cdots, \beta_{nn}$ interior to C satisfy an asymptotic condition $\lim_{n\to\infty} \left|(z-\beta_{n1})(z-\beta_{n2}) \cdots (z-\beta_{nn})\right|^{1/n} = \Psi(z)$ uniformly on any closed limited set exterior to C. Let the function $f(z)$ be analytic on and within C but not throughout the interior of B. Let $f_n(z)$ be the function analytic interior to B interpolating to $f(z)$ in the points $\beta_{n1}, \cdots, \beta_{nn}$ the least upper bound of whose modulus interior to B is a minimum. Then the sequence $f_n(z)$ converges in a characteristic way to $f(z)$ on C and even in certain points interior to B exterior to C (overconvergence). Compare for similar results a recent paper by the writer, Proceedings of the National Academy of Sciences, vol. 24 (1938), pp. 477–486, especially Theorems 1 and 3. (Received March 10, 1939.)

251. L. F. Walton: *On the theory of ideal numbers.* Preliminary report.

This investigation is based on the ideal theory developed by B. L. van der Waerden and E. Artin for integrally closed domains of integrity satisfying the finite chain condition ("Teilerkettenbedingung") and whose prime ideals are not necessarily divisorless. In the domains of integrity which are the subject of the present study, the existence of a denumerably infinite set of higher prime ideals is postulated. By introducing a "quasi-containment" relation defined in terms of the "quasi-prime decomposition" of the above ideal theory, it has been possible to imbed these domains in rings whose elements exhibit many of the properties of the Prüfer-Neumann ideal numbers. In particular, one obtains a unique additive decomposition in terms of p-adic components. (Received March 16, 1939.)

252. J. A. Ward: *Theory of analytic functions in linear associative algebras.* Preliminary report.

By means of the regular representations of an algebra, generalizations of the Cauchy-Riemann differential equations are obtained in terms of which analytic functions are defined. The usual properties of analytic functions are easily seen to follow, including results of Scheffers, Hausdorff, Ketchum, and Ringleb. The theory for commutative algebras is particularly simple. If $y=y_1e_1+\cdots+y_ne_n$ is an analytic function of $x=x_1e_1+\cdots+x_ne_n$, the jacobian matrix $\partial(y_1, \cdots, y_n)/\partial(x_1, \cdots, x_n)$ is the matrix representation of the derivative of y with respect to x. (Received March 16, 1939.)

253. F. J. Weyl: *Exponential curves.*

A normal exponential curve shall be defined by setting up $x_i=e^{\lambda_i z}$ in projective complex n-space $\{x_0, x_1, \cdots, x_n\}$, where the parameter z varies over the finite z-plane and the λ_i are distinct but arbitrary complex numbers. To this curve the theory of meromorphic curves (Hermann and Joachim Weyl, *Meromorphic curves*, Annals of Mathematics, (2), vol. 39 (1938), no. 3) is applied. Since the order $T(r)$ of such a curve, except for a bounded term, is equal to $r/2\pi$ times the circumference of the smallest convex polygon containing all points λ_i in the complex number plane, the second and third main theorems of this theory lead to certain results concerning circumferences of smallest convex polygons containing a given set of points in the plane. Similar considerations are carried through for the projection $y_j=\sum_{i=0}^{n}a_{ji}e^{\lambda_i z}$, $(j=0, 1, \cdots, m; m\leqq n)$, of a normal curve into a space of lower dimensionality. (Received March 17, 1939.)

254. G. T. Whyburn: *On the existence of certain transformations.*

It is shown that in order for a compact locally connected continuum to be mappable onto the interval (0, 1) by a non-alternating interior transformation $f(x)$ so that $f(a)=0$, $f(b)=1$, it is necessary and sufficient that M be a cyclic chain $C(a, b)$ from a to b. It follows from this that for each y, $(0<y<1)$, the set $f^{-1}(y)$ separates M irreducibly between a and b into just two components. Also, if M is unicoherent, f will be monotone. Related questions, such as the mappability of graphs and dendrites into an arc by interior transformations and the mappability of locally connected curves into an arc by light interior non-alternating transformations, are also considered. (Received March 8, 1939.)

255. L. R. Wilcox: *A theorem on curves in a projective space.*

Let C be a curve in a projective space S_n of n dimensions, and let T: $S_2\subset S_3\subset\cdots\subset S_{n-1}$ be a sequence of subspaces of S_n, the dimension of S_k being k. This paper considers configurations generated in the P_i by intersecting them with the linear osculants of C. In particular, a sequence of curves C_k, $(k=0, \cdots, n-2)$, is inductively defined as follows: $C_0\equiv C$; C_k is the set of intersections with S_{n-k} of the tangent lines to C_{k-1}. It is shown under mild hypotheses that C_k is the set of intersections with S_{n-k} of the osculating k-spaces of C. Special attention is devoted to the case when T is the sequence of linear osculants of C at a fixed point. (Received March 17, 1939.)

256. Max Wyman: *Non-holonomic covariant vector fields.*

In discussing covariant vectors fields for a Hausdorff space H, with coordinates

in a Banach space E, Michal (*Abstract covariant vector fields in a general absolute calculus*, American Journal of Mathematics, vol. 59 (1937), pp. 306–314) points out the advisability of postulating an inner product $[x, y]$ for the space E. In the non-holonomic case the author considers in addition to H and E another Banach space E_1, and takes $\overline{V}(\bar{x}) = M(x, V(x))$ to be the law of transformation of a non-holonomic contravariant vector field, where $M(x, y)$ is a solvable linear function of y on EE_1 to E_1, with $N(\bar{x}, y)$ as inverse. To introduce non-holonomic covariant vector fields he also postulates an inner product $[v, w]$ for E_1, and assumes $N(\bar{x}, y)$ has an adjoint $N^*_{(\bar{x})}(\bar{x}, w)$. The law of transformation of covariant vector fields is $\overline{W}(\bar{x}) = N^*_{(\bar{x})}(\bar{x}, w)$ and that of a covariant linear connection is $\overline{L}(\bar{x}, \overline{w}, \bar{\xi}) = N^*_{(\bar{x})}(\bar{x}\, L(\lambda\ w, \xi)) + N^*_{(\bar{x})}(\bar{x}, w; \xi)$. Under these definitions it is found that the usual tensor theorems can be proven, and one can introduce a normal representation theory for the covariant case. These results are in the main generalizations of results obtained by Michal and Botsford (Annali di Matematica, vol. 12, pp. 13–32) for the n-dimensional case. (Received March 17, 1939.)

257. Max Wyman: *Postulates for the determination of a non-holonomic linear connection.*

The author considers in non-holonomic tensor analysis a Hausdorff space H with coordinates in a Banach space E, and in addition another Banach space E_1. In this paper H is taken to be a generalized Riemannian space as defined by Michal (Paris Comptes Rendus, vol. 205 (1937), no. 14, p. 552). The existence of two functions $R(x, v)$ on EE_1 to E_1, and $\gamma(x, w)$ with the following properties, (i) $R(x, v)$ has the same properties as the metric form $g(x, \xi)$ of E, and (ii) $\gamma(x, w) = 0$ has only a finite number of linearly independent solutions, is assumed. It is then possible to state three postulates by means of which one can calculate a unique linear connection $K(x, v, \xi)$ in terms of $R(x, v)$, $\gamma(x, w)$, and r arbitrary covariant vector fields. These results are generalizations of results obtained for the n-dimensional case by Michal and Botsford (Annali di Matematica, vol. 12, pp. 13–32). (Received March 7, 1939.)

258. Max Zorn: *Menger-Brouwer dimension in subanalytic spaces.* Preliminary report.

The author has introduced the subanalytic spaces in connection with the theory of discontinuous groups. In this paper the theory of dimension in these spaces is examined. It is conjectured that the essential theorems of Menger's theory hold even when the concept "arbitrarily small" is defined by means of an arbitrary, not necessarily finite, covering of the subanalytic space by open sets. (Received March 18, 1939.)

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